FFT Acquisition of Periodic, Aperiodic, Puncture, and Overlaid Code Sequences in GPS

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BIOGRAPHY

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ABSTRACT

This paper presents a study of FFT-implemented circular correlation and its application to fast direct acquisition of GPS codes. This includes the periodic C/A-codes, practically non-periodic P(Y)-codes, never-repeating cryptographic M-code, and puncture acquisition (PA) codes which have been proposed to aid direct M acquisition as well as overlaid codes which are created by surface-reflected GPS signals extended beyond one code chip. FFT operates on blocks of incoming and replica code samples, thus providing simultaneous search over the entire block of code phases. It is straightforward to work with periodic codes for circular correlation. However, it is not obvious for puncture codes and long codes in particular. One major concern is how to ensure that the incoming and replica code samples contained within the working block could be correlated. In addition, it is quite possible that the data bit sign may reverse in the middle of an integration interval. Furthermore, how to efficiently make use of complex FFT when the data length is not a power of two or highly composite is critical for practical implementation. These design and computation issues are properly formulated in this paper and pertinent acquisition schemes are suggested.

INTRODUCTION

At the heart of a GPS receiver is the correlation performed between a segment of the incoming signal samples and a locally generated code replica. A successful correlation identifies the particular GPS satellite from which the signal originates while rejecting all other codes present at the same time. The correlation integration, acting as a matched filter with a large processing gain, de-spreads the signal and produces an enhanced signal to noise ratio (SNR) at the output for detection, delay and Doppler error discrimination, and navigation message data bit extraction. Reviews of GPS technology and its applications can be found in [Parkinson and Spilker, 1996; Kaplan, 1996].

In most GPS receivers, code correlation is implemented in the form of a digital correlator or a matched filter made up of basic logic circuits such as an XOR (exclusive-or) gate followed by an adder (integrate and dump), where the signal is quantized into one bit or two. Though being simple, the operation of a time-domain hardware correlator cannot run faster than the rate at which the signal samples arrive. Although a block of signal samples is integrated per correlation, only one code phase is tested each time, unless numerous hardware correlators are put in parallel, each assigned to a different code phase.

There has been a growing interest in software GPS receivers for flexibility gained from programmability. In most software GPS receiver designs, the code correlation is implemented via Fast Fourier Transform (FFT) [van Nee and Coenen, 1991]. Actually, FFT can be employed not only as a fast means to calculate the correlation function but also as a tool to perform domain conversions from time to frequency and vice versa. Once in the frequency domain, the incoming signal spectrum can be scrutinized for detection and suppression of narrowband interference [Peterson et al., 1996] and for simple translation to the baseband with Doppler removed in the process [Yang, Vasquez, and Chaffee, 1999a]. A selective processing can also be applied to a portion of the signal spectrum of interest, be it the upper sideband, lower sideband or both of the split-spectrum of the proposed GPS modernization signal.

This paper presents a study of FFT-implemented circular correlation and its application to fast direct acquisition of GPS codes. This includes the periodic C/A-codes, practically non-periodic P(Y)-codes, never-repeating cryptographic M-code, and puncture acquisition codes which have been proposed to aid direct M acquisition [Barker et al., 2000]] as well as overlaid codes which are created by surface-reflected GPS signals extended beyond one code chip [Yang, Muskat, and Garrison, 2000].

Operating on blocks of incoming and replica code samples, FFT thus provides simultaneous search over the entire block of code phases. It is straightforward to work with periodic codes for circular correlation. However, it is not obvious for puncture and particularly long code sequences. One issue is to ensure that the incoming and replica code samples contained within the working block could be correlated. In addition, it is quite possible that the data bit sign may reverse in the middle of an integration interval. Furthermore, it is critical to implement a complex FFT when the data length is not a power of two or highly composite.

To address these computation and design issues, the paper is organized as follows. First, linear and circular correlations are described with emphasis placed on their differences as implemented by FFT. Computational issues such as sampling rate, commensurability ratio, zero padding, and complex FFT are then discussed. The paper next focuses on FFT acquisition of periodic and aperiodic codes with four algorithms outlined. It then analyzes FFT acquisition of puncture and overlaid codes. Finally, concluding remarks are provided with a summary.

LINEAR VS CIRCULAR CORRELATION

Given two sequences of data points of length N, they will be successively shifted relative to each other to produce the values of their entire cross correlation function. There are two different ways to conduct the shifting, leading to either a linear or circular correlation accordingly.

In the linear correlation, the second sequence is shifted with respected to the first and zero-padded or truncated wherever the sequences do not line up. To illustrate, two sequences represented by the rectangles A and B are shown in Figure 1(a). The resulting linear correlation function, being the overlapped area, has a support of length 2N-1.

In the circular correlation, however, the second sequence is time-reversed circularly shifted with respect to the first, as shown in Figure 1(b), where two pseudo random number (PRN) sequences are represented by the triangles with the height to indicate the phase. It is important to note that the resulting circular correlation function has a support of length N.

FFT operates on a block of samples of length N and implicitly assumes that the signal is of periodicity of N. Because of this, FFT-implemented convolution or correlation is circular in nature. If non-periodic finite signals are of interest, extra care has to be given to the use of FFT for linear convolution or correlation. Otherwise, there will be a time-aliasing due to undersampling of the correlation function [Kunt, 1986]. To perform linear correlation with FFT, both the sequences have to be zero-padded to at least a length of 2N-1.

This is consistent with periodic C/A codes when the correlation interval is the code epoch of 1ms. In fact, the correlation between two periodic sequences over one period, even implemented in a linear manner, is circular and it is straightforward to apply FFT.

However, there are a number of circumstances where one actually faces the problem of linear correlation and it has to be converted into a circular one in order to benefit from the fast computations offered by FFT. This aspect will be further addressed in the subsequent sections.

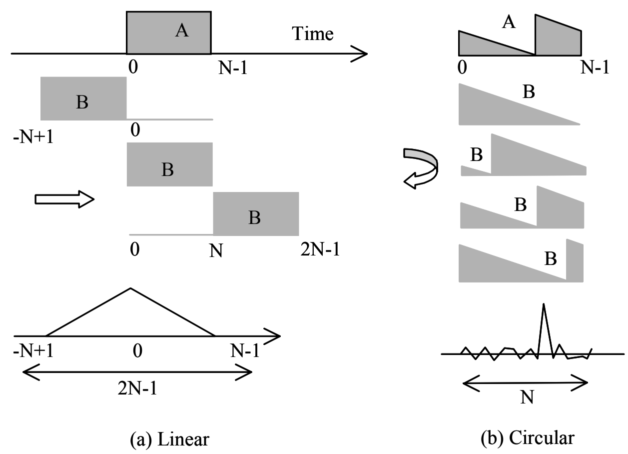

Figure 1. Linear vs. Circular Correlation

SAMPLING RATE AND ZERO-PADDING

Modern GPS receivers digitize the analog GPS signal at a suitable sampling rate with a suitable number of quantization levels. For a binary sequence with rectangular waveform at a chipping rate of f_c (chip per sec or cps), its fundamental frequency is $f_c/2$ (Hz) whereas the first null of its spectrum is at f_c (Hz). Since the signal may be band-limited by the receiver front-end within its first nulls, the Nyquist sampling rate is taken as $f_s > 2f_c$.

In practice, however, a sampling rate much higher than this minimum Nyquist rate is typically used. Its selection is determined by both the last stage analog IF frequency and the initial digital IF frequency. The former is related to the analog front-end down-conversion frequency plan design while the latter entails many considerations. An oversampling allows for pipelined matched filters with closely spaced correlations to operate for rapid acquisition. It may also be necessary in order to apply well-behaved digital filters in order to relax requirements on front-end analog filters, which are more expensive for stable operation over a large range of environmental conditions.

For an FFT-implemented correlation, the sampling rate not only determines the size of each FFT per operation block (so is the computational burden) but also the resolution of the correlation function at the FFT output as well as the theoretic accuracy of code matching.

In a conventional hardware implementation, how many times (typically three, known as the early, prompt, and late correlators) and where (around the correlation peak) the underlying correlation function is evaluated is determined by the number of correlators available per channel and the code phase spacing between individual correlators. The correlator spacing can also be viewed as the resolution at which the correlation function is sampled.

In contrast, the entire correlation function is produced by FFT and it has a resolution being the inverse of the sampling rate, i.e., $1/f_s$ (or f_c/f_s in terms of chips). Increasing the sampling rate can improve the correlation resolution or correlator spacing for a better code tracking performance. In addition, a high-resolution correlation provided by FFT can be used to drive a multipath estimator for multipath mitigation. However, this inevitably increases the computational burden of FFT per operation block.

The accuracy of code matching, that is, how closely the chip transitions of two code sequences can be aligned together without ambiguity, is also determined by the sampling rate in relationship to the chipping ratio, known as the commensurability ratio [Thomas, 1988]:

$$\gamma = \frac{f_s}{f_c} = \frac{n_s}{n_c} \tag{1}$$

where n_s and n_c are integers with all common factors cancelled (relative prime).

The commensurability ratio indicates that over an exact amount of n_c code chips, an exact amount of n_s samples can be taken. If all these n_c chips are piled up on top one another with the leading edges to leading edges and trailing edges to trailing edges, the corresponding sampling points fall onto the piled chips with a spacing of n_c/n_s of a chip. If either the samples or the code chips are moved in one direction or the other by this amount, the resulting correlation between the two will not change. This is an ambiguity in code matching due to discrete sampling and is also the lower bound in accuracy one can expect [Yang, 1996]. In practice, however, the commensurability is not constant due to the Doppler frequency shifting in the incoming signal and the instability in the local sampling clock.

Without constraint, it is ideal to select such a sampling rate that the number of data points in an FFT interval is a power of two or four to attain the maximum efficiency of FFT or highly factorable so that a mixed radix FFT can be applied. If neither is possible, one has to implement an FFT-based correlation at an arbitrary sampling rate.

At a first glance, one may be able to apply a radix-2 FFT to an arbitrary data sequence padded with zeros to its nearest power of two or four. This technique seems to work when the number of padded zeros is small and that the two sequences are almost aligned as in the tracking mode, as illustrated in Figures 2(a) and 2(b) for head and tail matches, respectively.

However, the zero padding to a power of two or four may create partial correlation with reduced SNR, depending on where either sequence starts. This is illustrated in Figure 2(c) for a partial match case with two ambiguous peaks.

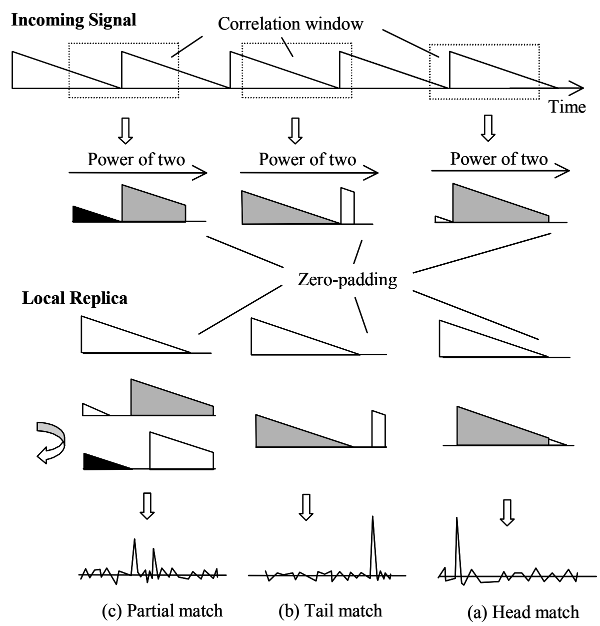

Figure 2. Single Length Zero-Padding FFT

When one has to deal with an arbitrary sampling rate that results in a not-highly-factorable number of data points per FFT, the technique by [Stockham, 1966] can be applied as the last resort rather than the brute force DFT.

The technique employs the idea that new data sequences are constructed to a power of two in length and their circular correlation with FFT is produced without degradation. This is shown in Figure 3, where the length of the new sequences is the smallest power of two above 2N-1 as

$$N_1 = (2^n)_{min} \geq 2N\text{-}1 \quad \text{for an integer n} \qquad (2)$$

The circular correlation over the extended sequence N_1 may produce multiple peaks. However, if the circular correlation is restricted to the first N shifts, the resulting values are unique and are what we are actually looking for. This restriction eliminates the computations otherwise wasted for the unnecessary shifts.

The same procedure can be modified to perform a linear correlation by FFT in which two N-sequences are zero-padded to N_1 as given in Eq. (2) without placing the duplicated replica code from N_1-N+1 to N_1

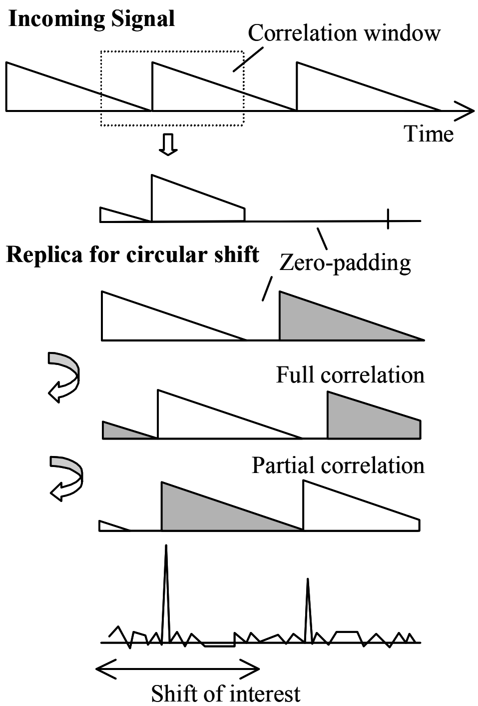

Figure 3. Double-Length Zero-Padding FFT (Method 1: Circular Correlation at Arbitrary Length)

COMPLEX FFT

Inside Fourier transform are complex operations involved (multiplications and additions), although it can be decomposed into a pair of real sine and cosine transforms. As a result, an FFT operating on complex data is more efficient than on real data of the same length. It is always recommended to work with complex FFT applied to complex data.

When only real data are at hand, the following steps can be taken to reduce the overall computations. First, one can pair two real sequences, say, x(n) and y(n), into a complex one as z(n) = x(n) + jy(n). Then apply the complex FFT to the complex sequence as Z[k] = FFT{z(n)}. Finally recover the respective spectra for the real sequences X[k] = FFT{x(n)} and Y[k] = FFT{y(n)} from the complex sequence spectrum Z[k]. The complete procedure can be found in [Smith and Smith, 1995].

In GPS applications, unless the incoming signal and the local replica are made complex in the frequency down and up translation processes [Yang 2000], they are real in nature. Nevertheless, there are at least two ways to make use of complex FFT for their processing. The first way is to pair two consecutive segments of the incoming samples, padded with zeros if necessary, into a complex segment, as illustrated in Figure 4. The other way is to pair one segment of incoming samples with one segment of code replica into a complex one and then to follow the process depicted in Figure 4.

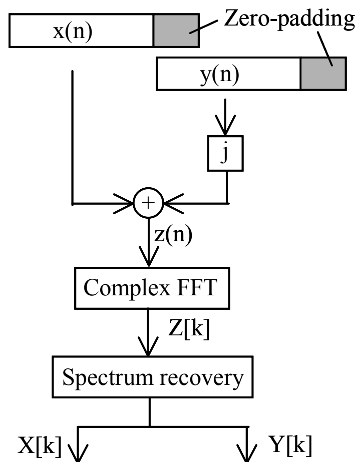

Figure 4. Complex FFT on Pair of Real Sequences

PERIODIC CODES

By design, the GPS coarse acquisition (C/A) codes are short in duration (1023 chips over 1ms), of relatively low chipping rate (1.023Mcps), and repetitive (every 1ms). This makes C/A codes suitable for direct search acquisition and it is so implemented in almost every GPS receiver today.

With hardware correlators, the incoming signal samples are correlated, for each integration interval, with a local set of replica samples whose starting sample is sequentially displaced by a certain amount (typically half chip a time) across the timing uncertainty interval to look for a match. The actual operation, though being linear in implementation, behaves as if it is circular when the integration interval is about one C/A code epoch because of the periodicity.

This periodicity also makes the circular correlation of FFT directly applicable to C/A codes on an epoch-to-epoch basis. According to the signal specs, the 1ms integration of C/A codes should provide the necessary processing gain for a correct detection, which may be further cumulated over time for better estimation. In preparation of the local replica for circular correlation with FFT in each epoch, it is important to sample the local replica from the very beginning. This is because this 1ms boundary can be the 20ms boundary (i.e., the navigation data bit) and can also be the 1s boundary, all needed for later data sync and other timing purposes.

Without knowing the 1ms boundary in the search mode, it is quite possible that a data bit sign may reverse itself within a correlation window (once every 20ms at most). If it happens, it can reduce the correlation peak and in the worst case even destroy an otherwise perfect correlation. As shown in Figure 5, the data bit sign reversal within a correlation window makes it no longer periodic. By consequence, the 1ms circular correlation cannot be used alone to decide the absence of a signal.

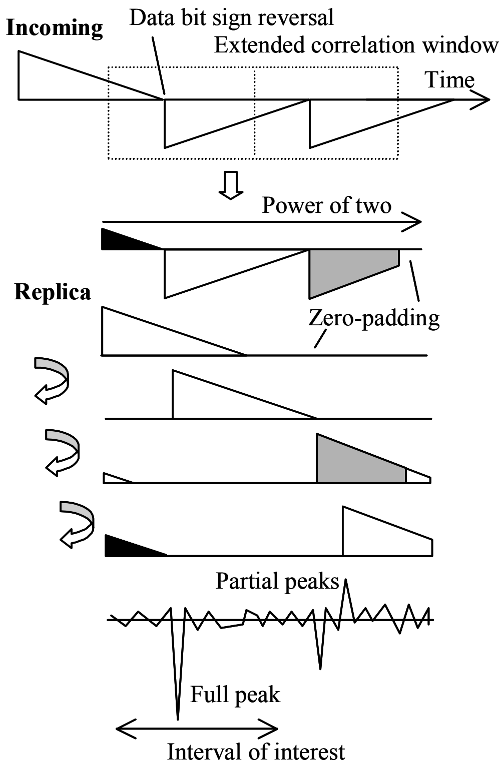

Figure 5. Double-Length Zero-Padding FFT
(Method 2: Linear Correlation at Arbitrary Length)

Two solutions are available to this problem. One is to conduct at least two 1ms circular correlations before deciding the absence of a particular code of interest in the signal. The other is to conduct a 2ms linear correlation as shown in Figure 5.

For the case shown in Figure 5, if the circular correlation by FFT is carried out completely, there are three correlation peaks. The major peak in the first half of shifts (i.e., to delay the replica to align with the incoming signal) corresponds to the full correlation, while the two minor ones in the second half (i.e., to advance the replica to align with the incoming signal) correspond to partial correlations. For practical purposes, we are only interested in the first half of shifts. By restricting this shifting range, we also reduce the computation load.

Another advantage of zero padding to double-length as shown in Figures 3 and 5 is a finer frequency resolution associated with the apparent increase in time interval. It thus permits frequency-domain Doppler removal with smaller frequency steps [Yang, 2001c; 2001d].

Further, the procedure depicted in Figure 5 can be viewed as another way to perform a zero-padded FFT for an arbitrary sampling rate, alternative to the approach described in Figure 3. In the absence of data bits, a third way to perform FFT for an arbitrary sampling rate, zero-padded to a power of two, is to simply switch the incoming signal and the replica in their role as shown in Figure 5. This will not introduce the unwanted artifacts as illustrated in Figure 2.

LONG AND APERIODIC CODES

GPS precision code or its encrypted version, denoted by P(Y)-code, is very long (exactly one week), with a high chipping rate (10.23Mcps), and practically non-repetitive. The proposed GPS modernization signal M-code is cryptographic in nature and will never repeat. Both P(Y)-code and M-code were not originally designed for direct acquisition and their acquisition rather relies upon the handover from an acquired C/A-code. The latter reduces the time uncertainty to within a fraction of a C/A-code chip, thus rendering the post-handover search of P(Y)-code or M-code possible.

However, direct acquisition of P(Y)-code and M-code becomes indispensable when the C/A-code is totally jammed, spoofed, or disabled. Unless guided by an ultra stable atomic clock, the conventional sequential "point" search (one code phase per pre-detection interval at a time) is too slow particularly under large time and frequency uncertainty. Block search, which tests on a large number of code phases and Doppler bins simultaneously in parallel, on the other hand, becomes the preferred choice. Both "hard" and "soft" parallelisms have been proposed. The adaptation of FFT from periodic to very long code sequences is just an example of the attempts to soft parallelism [Yang, Vasquez, and Chaffee, 1999b].

For a long or an aperiodic code sequence, one can only work with a subsequence of it. If the data bit sign reversal is out of concern, one can freely select the number of samples per correlation so that a radix-2 or 4 FFT can be applied directly. A typical design procedure takes the following steps [Yang, 2001a]. First, one selects the length of incoming signal segment, M, long enough for the correlation integration to produce the needed processing gain under normal signal conditions. One then selects the length of replic segment, N $\gg$ M, generated around the estimated time encompassing the entire uncertainty interval as a search window large enough to surely contain the incoming signal segment. Finally, one implements a search algorithm that can efficiently find the incoming signal block M out of the extended replica segment N. Several search algorithms are described below.

A mathematical model is first established with the help of Figure 6. The true signal time t_S is assumed to surely fall within the time uncertainty interval T_U around the receiver time estimate t_R. To find where this unknown t_S lies within the uncertainty interval T_U, a segment of incoming signal starting at t_S will be correlated successively with replica segments whose starting point, denoted by t_i, is selected somewhere within T_U. The starting point of the replica segment that produces the maximum correlation exceeding a threshold will be taken as the best estimate of t_S.

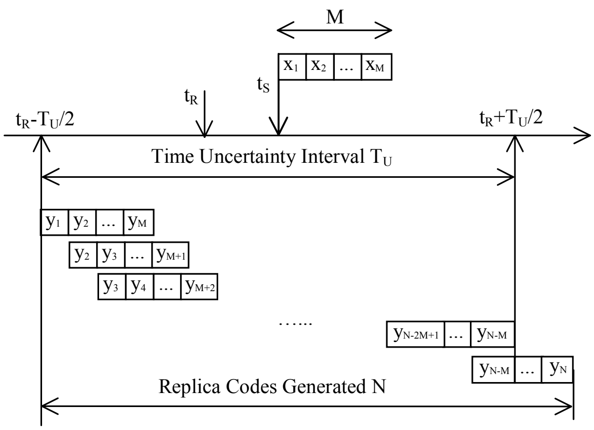

Figure 6. Direct Acquisition under Large Uncertainty

As shown in Figure 6, the incoming signal samples can be collected into a vector $X = [x_1, x_2, \ldots, x_M]$. In order to cover the entire uncertainty interval, the total number of samples generated for the code replica is $N = T_U f_s + M$, where f_s is the sampling rate. Putting all replica samples into a vector gives $Y = [y_1, y_2, \ldots, y_N]$. In total, there are N-M segments of replica within the uncertainty interval, each starting at a different sample. Denote each replica segment starting at t_i by $Y_i = [y_i, y_{i+1}, \ldots, y_{i+M-1}]$ that will be correlated with the signal segment X. The resulting correlations over the entire uncertainty interval can also be put into a vector $Z = [z_1, z_2, \ldots, z_{N-M}]$, where z_i is the i-th correlation between Y_i and X.

The above operations can be put into the matrix form as given in Eq.(3). In Eq.(3a), the replica matrix is generated by augmenting each segment vector in each row to the full replica vector in a circular manner. The dimensions are thus changed from (N-M)×M to (N-M)×N. The X vector is also zero-padded with its dimension increased from M to N.

Eq.(3b) is an alternative representation in which an incoming signal matrix of dimensions (N-M)×N is generated by first zero-padding the X vector from M to N and then consecutively shifting it in a circular manner for each row. The replica vector is just Y for all the replica codes generated in the uncertainty interval.

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{N-M} \end{bmatrix} = \left[\begin{array}{cccc|ccc} y_1 & y_2 & \ldots & y_M & y_{M+1} & \ldots & y_N \\ y_2 & y_3 & \ldots & y_{M+1} & y_{M+2} & \ldots & y_1 \\ \vdots & \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ y_{N-M} & y_{N-M+1} & \ldots & y_N & y_1 & \ldots & y_{N-M-1} \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3a}$$

$$= \begin{bmatrix} x_1 & x_2 & \ldots & x_M & 0 & 0 & \ldots & 0 \\ 0 & x_1 & \ldots & x_{M-1} & x_M & 0 & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & 0 & 0 & x_1 & \ldots & x_M \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \\ y_{M+1} \\ \vdots \\ y_N \end{bmatrix} \tag{3b}$$

Scheme 1: Full Interval Correlation (Incoming Signal Augmentation)

The ideal solution is to evaluate the full interval correlation as given in Eq.(3). It amounts to N-M correlations of length M and the total number of multiplications involved is on the order of $(N\text{-}M)\times M^2$. With hard parallelism, the full interval correlation can be calculated with N-M correlators at once or with a smaller number of correlators several times. Similarly, it can also be calculated with parallel matched filters.

The full interval correlation can be implemented with soft parallelism. To see this, denote the zero-padded incoming code vector X by $\overline{X}$. The Z vector of circular correlation between $\overline{X}$ and Y can be calculated with FFT in one of the following two ways:

$$Z = Y \otimes \overline{X} = IFFT\{FFT\{\overline{X}\} \circ FFT^*\{Y\}\} \tag{4a}$$

$$= \overline{X} \otimes Y = IFFT\{FFT\{Y\} \circ FFT^*\{\overline{X}\}\} \tag{4b}$$

where ⊗ stands for circular correlation, ° multiplication per frequency bin, and * complex conjugate.

Given the proper choice of M and N, this full interval search will reach the correct solution for a single incoming segment. The number of multiplications required for calculating all correlations changes from $(N\text{-}M)\times M^2$ to $2N\log_2(N)+M\log_2(M)+N$, thus producing considerable computation-saving when N and M are large. The pruning FFT can be used to cut off those butterfly branches corresponding to zero-padded inputs, further reducing computation load [Yang, 2001b].

The full interval correlation offers a deterministic behavior. However, taking on the full uncertainty interval in a single block requires computation that may be formidable even with FFT, although it may be useful for near real-time or post-processing. By the random nature of timing uncertainty, the true value may lie anywhere in the interval and the full search from one end to the other undiscriminatorily seems wasteful.

Scheme 2: Extended Replica Folding

Instead of augmenting the incoming code segment from X of small M to $\overline{X}$ of large N as in Eq.(3b), an alternative technique for simultaneous search over the entire uncertainty interval is to reduce the local code replica from Y of large N to a vector of small M. This is the extended replica folding technique described in [Yang, Vasquez, and Chaffee, 1999b; Yang et al., 2000].

To illustrate, a simple example is shown in Figure 7. An incoming signal segment is assumed to have four chips denoted by *a*, *b*, *c*, and *d*, from time *t* to *t+T*. The arrival time of the incoming signal is only known by an estimate, $\hat{t}$, with uncertainty Δ. To cover the entire uncertainty search interval, the local code is generated from $\hat{t}$ -Δ to $\hat{t}$ +Δ+*T*. As shown, the extended code replica has five segments and each segment has four chips (a_i, b_i, c_i, and d_i, for $i = 1,2,3,4,5$). The five segments are then folded into a single segment with four new chips being given by

$$\bar{a} = a_1 + a_2 + a_3 + a_4 + a_5 \tag{5a}$$

$$\bar{b} = b_1 + b_2 + b_3 + b_4 + b_5 \tag{5b}$$

$$\bar{c} = c_1 + c_2 + c_3 + c_4 + c_5 \tag{5c}$$

$$\bar{d} = d_1 + d_2 + d_3 + d_4 + d_5 \tag{5d}$$

The folded replica segment is then circularly correlated with the incoming segment, resulting in four possible correlations between (*a*, *b*, *c*, *d*) and ($\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$), ($\bar{b}$, $\bar{c}$, $\bar{d}$, $\bar{a}$), ($\bar{c}$, $\bar{d}$, $\bar{a}$, $\bar{b}$), and ($\bar{d}$, $\bar{a}$, $\bar{b}$, $\bar{c}$), respectively. If the cross-correlation among the sequences is ideally zero, a correlation peak occurs at the third position in the folded segment. The matched replica (c_3, d_3, a_4, b_4) can then be distinguished from other three sequences (c_1, d_1, a_2, b_2), (c_2, d_2, a_3, b_3), and (c_4, d_4, a_5, b_5) by simple correlation check.

The extended replica folding technique introduces a self-inflicted loss of SNR due to folding and an ambiguity in folds that needs to be resolved. However, the length of FFT used in the extended replica folding technique is much shorter than that of the full interval correlation technique. When the ratio of the number of chips in each segment M over the number of segments N is large in the order of hundreds and even thousands, computational advantage is pronounced: it reduces the number of correlations from the order of MN to the order of M+N.

Scheme 3: Overlap and Add (Superposed Addition)

There are two classic approaches to split convolution between two sequences, one being much longer than the other [Kunt, 1986]. The first is the overlap and add (or superposed addition) technique to be described in this section and the other is the overlap and discard (or juxtaposition) technique to be presented in the next section. Both are adapted here for use in our GPS applications for correlation.

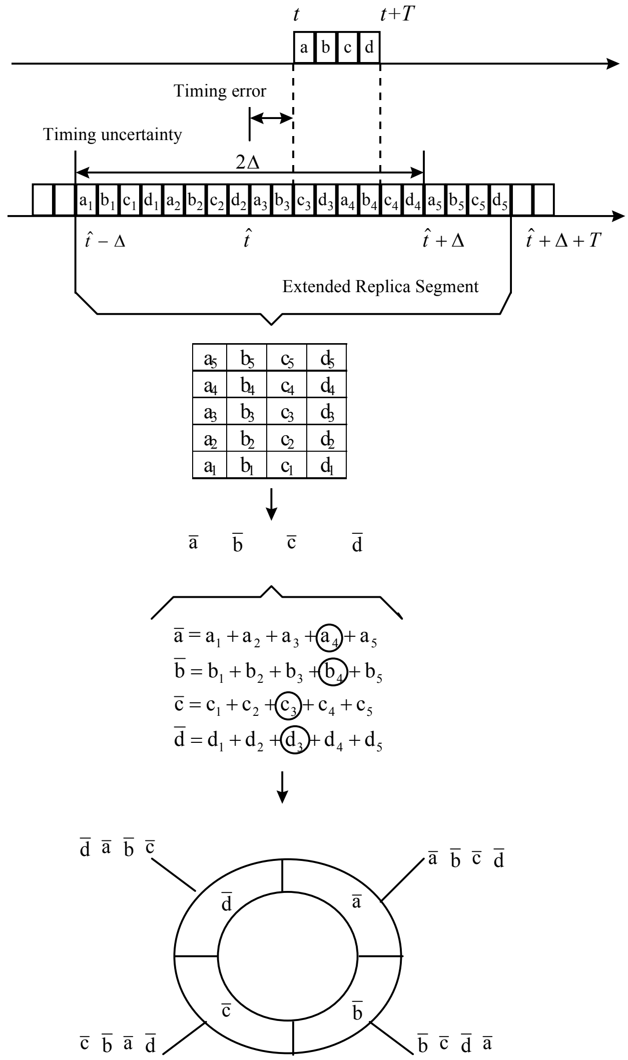

Figure 7. Extended Replica Folding Technique

As shown in Figure 8, the extended replica sequence is divided into small segments of length L. Each segment L is then pre-appended with N-L zeros so that the incoming segment of length M do not overlap (i.e., N-L > M) when using an N-point FFT. It is also possible to post-append zeros to the replica segment.

The circular correlation function produced by FFT for each segment has three different sections: the first M-1 and last M-1 points represent partial correlations while the middle N-2M-2 is the full correlation.

The last section of M-1 points of one segment can be combined by addition with the first section of M-1 point of the next segment immediately behind it to reconstruct the full correlation for this section. The partial correlation sections in consecutive segments overlap and they add up to the full correlation, hence the name "overlap and add".

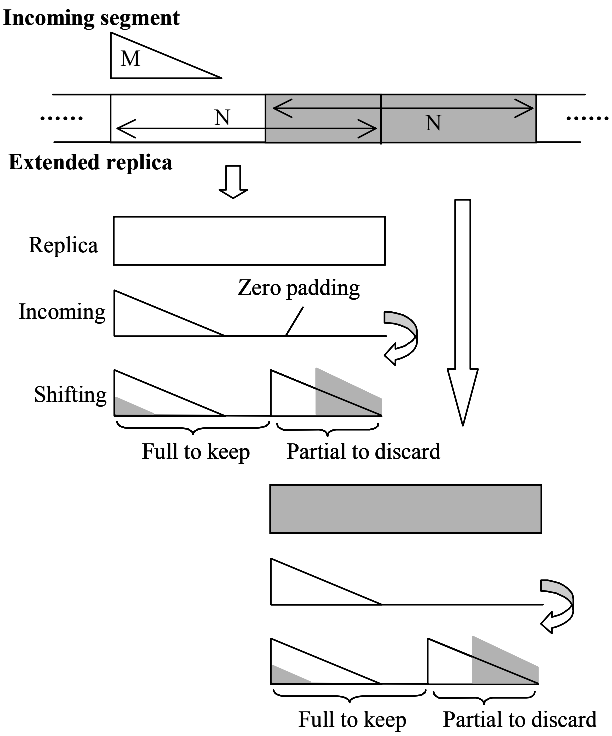

Figure 8. Overlap-and-Add Algorithm

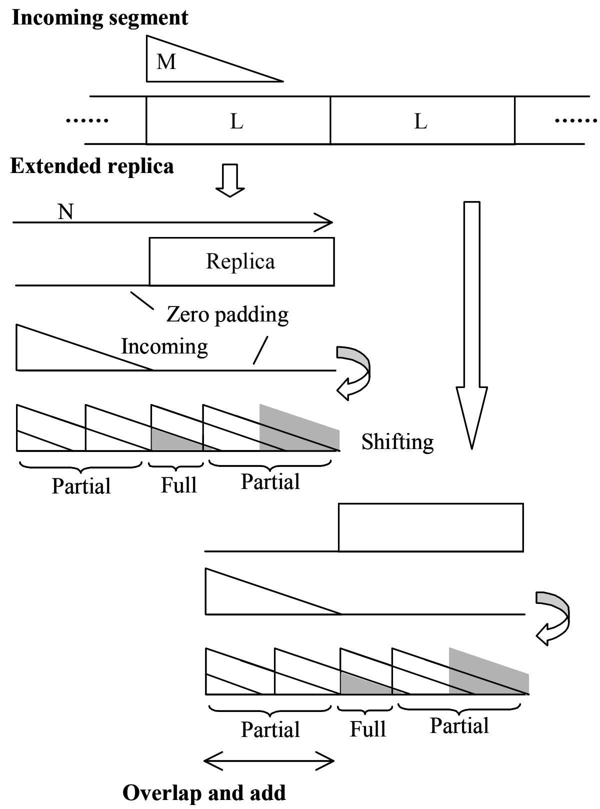

Figure 9. Overlap-and-Discard Algorithm

Scheme 4: Overlap and Discard (Juxtaposition)

The overlap-and-discard algorithm described in this section is adapted from the overlap-and-save algorithm used for convolution with FFT. It differs from the overlap-and-add algorithm of the last section in that, rather than padding with zeros, it overlaps the data sequences within the circular FFT. The overlapped portion is to be discarded, hence the name.

As shown in Figure 9, two consecutive replica segments overlap by M-1 points. At the output of the FFT correlation of the first segment, the first N-M+1 points are full correlations to keep whereas the last M-1 points are partial correlations with unwanted sum to discard. The full correlations for these discarded M-1 points in this segment are later on supplied by the overlapped M-1 points in the second segment.

When N = 2M plus a number of zeros padded to a power of two or four, this overlap-and-discard algorithm reduces to the double length zero-padding FFT scheme shown in Figure 5 for linear correlation at arbitrary length.

The above four schemes all work on a single incoming signal segment and an extended replica segment. If the SNR is high enough and the computation can be completed within the time line, the direct instantaneous acquisition is achievable.

However, the number of code phases that can be included within a single segment and thus searched simultaneously cannot be increased unlimitedly due to physical constraints. When the uncetainty is large, not all code phases can be covered at once and multiple segments have to be used. In addition, for weak signals the coherent predetection integration interval cannot be increased unlimitedly either because of unknown Doppler frequency or otherwise the deadtime between integrations is excessive. This is also true when a large number of integrations is accumulated over time to produce a processing gain sufficient to burn through a heavy jamming. These situations necesitate a sequential search of multiple incoming signal segments with their corresponding extended replica segments over time.

The four schemes described above can be used as a correlation engine in sequential block search. To do so, a bidirectional strategy is suggested in [Yang, 2001a], which manages multiple incoming signal and replica segments in an efficient manner across a large uncertainty interval. Furthermore, its nonlinear coverage with memory places the search window in a distribution similar to the initial time estimate. As such, larger processing gains are given to more probable events. As a result, a higher probability of detection and a smaller mean time to acquisition are expected.

Another technique involving multiple incoming signal and replica segments is the polyphase filter [Abusalem and Harris, 1999], also called the weighted overlap and add algorithm [Crochiere and Rabiner, 1983]. It has been implemented for better frequency-domain processing in interference suppression among others.

PUNCTURE ACQUISITION (PA) CODES

GPS modernization signals set forth several aiding codes in transition to M-code acquisition [Barker et al., 2000]. One option is to puncture each never-repeating cryptographic code and fill in it with a directly acquirable code in such a manner that the replacement pattern carries the timing information.

Although a matched filter is a suitable hardware solution, the FFT-implemented circular correlation can accomplish the same task. As a reverse problem of looking for an incoming signal segment in the extended replica, the puncture acquisition now searches for a known sequence in the incoming signal stream.

However, the puncture code shows up in the incoming signal stream only once in a while. As a result, it has to be treated as an aperiodic code using linear correlation and to use an extended incoming segment so as to ensure the puncture code within the window whenever it appears.

The four FFT acquisition schemes described in the previous section can be applied for acquiring a puncture code. This is quite similar to the acquisition of a preamble for data sync in general communications but the FFT here implements soft parallel search in this case.

The technique can be extended to acquiring multiple short codes in different frequency bands simultaneously or hopping between these bands. Once these codes are acquired, their presence in the time-frequency map can be further processed to extract the timing information among others.

OVERLAID CODES

By overlaid codes, we mean the received signal is made up of many delayed and weighted copies of the same original code. Mathematically, the overlaid code s(t) can be written as

$$s(t) = c(t) + \sum_{k=1}^{K} \alpha_k c(t - \tau_k) + n(t) \qquad (6)$$

where $c(t)$ is the primary code, (α_k, τ_k) are the complex amplitude and delay experienced by the k-th code copy, and $n(t)$ is the noise. The number of copies K and their parameters are assumed to be constant over a brief time interval but vary over time.

We have identified three practical situations in which a code may lie over itself multiple times. The first situation is the well known multipath phenomena in which each delay τ_k is typically within one code chip while each weight $|\alpha_k|$ is less than one. Multipath degrades the code tracking performance because of the random time bias it introduces. It has been shown that narrow correlator spacing can improve code-tracking performance against multipath [van Dierendock, Fenton, and Ford, 1992]. A large amount of closely spaced correlators have also been used for multipath estimation and mitigation [Townsend, Fenton, and van Nee, 1995].

Equivalent to the hardware correlator spacing is the resolution of FFT-implemented correlation, which is related to the sampling interval. Since FFT correlation provides not only the entire correlation function but also its spectrum, it is suited for multipath estimation and mitigation so long as the correlation function is available with sufficient resolution. The correlation resolution can be improved by increasing the sampling rate. This, however, inevitably increases the computation load. An alternative way to increase the correlation resolution is to zero pad the correlation spectrum before taking its inverse FFT. Some spectral processing techniques may also be applied to mitigate multipath.

The second situation is the use of ocean-reflected GPS signals for geophysical remote sensing. Sea surface reflected GPS signals were first observed in surprise as undesirable [Auber, Bibaut, and Rigal, 1994]. The potential of these signals was quickly recognized [Katzberg and Garrison, 1996]. Water surface-reflected GPS signals can serve as a free illumination for bistatic remote sensing for geophysical study and for passive altitude determination. Recent progresses can be found in [Komjathy et al, 2001].

The ocean-reflected GPS signals can be viewed as a useful multipath in contrast to the harmful multipath discussed above. In this case, the reflected signals are primarily left hand circularly polarized (LHCP) compared to the right hand circularly polarized (RHCP) direct signals. The weights $|\alpha_k|$ are still less than one but some of the delays τ_k can be as large as a dozen of code chips. This requires careful selection of the segment length large enough to cover the extended code delay.

FFT can be used to produce the delay-Doppler map of the correlation power over the desired code chips and Doppler bins [Yang, Muskat, and Garrison, 2000]. In fact, FFT implementation is much more efficient than a hardware correlators-based approach. It can also be used to perform blind search for the reflected signals over a large time interval [Lowe et al, 2000].

The third situation is the presence of spoofing signals. In navigation warfare, the in-the-air GPS signals may be received, amplified, and rebroadcast by a stationary or mobile transmitter. Locally generated GPS look-alikes may also be aired. As a result, in the GPS signal bandwidth there may contain multiple copies of the same code for a given SVN, differing in time and Doppler, to confuse or overwhelm unprotected users.

The weights $|\alpha_k|$ associated with spoofing signals may be much stronger than the true signal and their delays τ_k can be positive (later) or negative (earlier). The use of an FFT-implemented circular correlation as described in this paper allows for easy detection and acquisition of the spoofing signals if present. It is then followed by tracking in pseudo ranges and range rates and ultimately in geocentric position to identify and locate the spoofers.

CONCLUSIONS

This paper presented a study of FFT-implemented circular correlation for direct acquisition of GPS codes, including periodic C/A-code, extra long P(Y)-code, never-repeating M-code, puncture acquisition code, and overlaid codes.

As circular correlation, FFT is directly applied to extended replica folding of long codes and periodic codes when navigation data bit does not reverse its sign in the midst of integration. All other codes, however, necessitate linear correlation. The overlap-and-add and overlap-and-discard algorithms were presented for that purpose.

Computation issues such as sampling rate, zero padding, commensurability ratio, and complex FFT were discussed. Selecting the FFT length for overlaid codes and zero padding to a power of two or to increase resolution were other practical yet important design aspects to consider.

Jamming or co-located RF interference is another serious problem for acquisition. Although not discussed explicitly, the FFT acquisition schemes presented in this paper are compatible with the frequency-domain jamming suppression and Doppler removal approaches [Yang, Vasquez, and Chaffee, 1999a]. The zoom and pruning FFT techniques [Yang, 2001b] can be employed in these acquisition schemes to further reduce computation load.

For weak signals or under wideband jamming, an extended coherent and/or incoherent integration may be employed to boost the signal to noise ratio. However, it requires an extra care to handle both the carrier and code Doppler effects as well as receiver clock errors during the prolonged integration. The cyclostationary property of GPS signals can also be exploited for this purpose. The present acquisition schemes can be extended to such weak signal cases for coherent and/or incoherent integration.

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