Acquisition Schemes for Software GPS Receiver

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BIOGRAPHY

David M. Lin received the B.S.E.E. from Tatung Institute of Technology, Taiwan, 1970, the M.S.E.E. and the M.E.M.E. from Tennessee Technological University Cookeville, Tennessee, 1977, 1978 respectively and the M.S.C.S. from Wright State University, Dayton, Ohio, 1984. From 1979 to 1985, he was a software Engineer at System Research Laboratories, Inc. Dayton OH. Since 1985 he has been an Electronics Engineer at the Air Force Research Laboratory, Wright Patterson Air Force Base, OH. His work involves Electronic Warfare, Digital Signal Processing, Electronic Warfare Instrumentation, and Radar and Electronic Countermeasure Simulation. He has received 3 patents.

James B.Y. Tsui was born in Shantung, China. He received the B.S.E.E. degree from National Taiwan University, Taiwan, the M.S.E.E. degree from Marquette University, Milwaukee, WI, and the Ph.D. degree in electrical engineering from the University of Illinois at Urbana, in 1957, 1961 and 1965 respectively. From 1965 to 1973, he was an Assistant Professor and then Associate Professor in the Electrical Engineering Department of the University of Dayton, Dayton, OH. Since 1973 he has been an Electronics Engineer at the Air Force Research Laboratory, Wright Patterson Air Force Base, OH. His work is mainly involved with microwave receivers and his recent research is on digital microwave receivers and global positioning system (GPS) receivers. He has written four books on microwave and digital receivers, two chapters for two books and published over 70 technical papers. He has received about 30 patents.

ABSTRACT

This paper discloses two new acquisition approaches to improve the efficiency of conventional acquisition methods used in GPS software receivers. The first single side band approach will reduce the process time and search time of the conventional approach by more than half. The second approach is an innovative real number implementation of the delay-and-multiply concept which was developed under complex numbers. Since this new approach uses real numbers instead of complex numbers, it reduces the processing time by more than half. Also, it makes the delay-and-multiply concept more feasible and practical. This paper also introduces a methodology to estimate the noise statistics and develop the threshold for detection. Based on this methodology, the performances of the two new approaches is analyzed, estimated by simulation, and compared against the performance of the conventional approach. The paper concludes with a demonstration of two new approaches with field collected data and recommends a potential application area where the new approaches can be applied.

INTRODUCTION

This paper presents two newly developed acquisition receiver approaches for acquisition of the Coarse/Acquisition (C/A) code of the Global Position System (GPS) and compares the performance of these approaches with the conventional approach which is widely used in software GPS receivers. The C/A code is a code division multiple access (CDMA) system where a unique signal is assigned to each satellite in the GPS system. In order to obtain the accurate pseudo range of the selected satellite, the starting time of the C/A code sequence in the received signal from the satellite must be accurately measured. The GPS receiver applies correlation to measure this timing. The received signal is correlated with the locally generated replicas of the selected satellite's signal. The traditional hardware GPS receiver acquires this timing by continuous sliding, multiplication, and addition. The C/A code for each satellite has 1023 chips and repeats itself every millisecond. For pseudo range accuracy, less than ½ chip sliding step is required. There are 2046×2046 multiplication and addition operations needed to align the C/A code. Due to the relative velocity between the satellite transmitter and the GPS receiver, Doppler effect will introduce ±10kHz of frequency uncertainty around the carrier frequency. In order to resolve the frequency to 1 kHz, 21 simultaneous 1kHz Doppler frequency bins are required. If the signal to noise ratio (S/N) is above a predefined threshold, the minimum number of multiplications and additions for a complete C/A code acquisition for a selected satellite is 21×2046×2046. If the S/N is less than the threshold or the number of samples per chip is increased (for better resolution), double or triple operations will be needed. To improve the efficiency, the software receiver conventional approach implements the correlation algorithm in the frequency domain. This is called the Discrete Fourier Transform (DFT) correlation method and will be discussed in the following section. If a Fast Fourier Transform (FFT) algorithm is used, this approach will need $21\times(2\log_2 2048+1)\times2048$ additions and half of the number of multiplications. Compared with the traditional hardware slide-and-multiply approach, the efficiency is improved exponentially in the number of operations. The two new approaches presented in this paper still use the DFT correlation method but the purpose of the first approach is to reduce the number of points for calculation. The purpose of the second approach is to eliminate all Doppler bins. The details of these approaches will be discussed in later sections. One or two simulated satellite signals and one random noise signal which represents the background noise are applied to the different acquisition approaches to calculate the noise and signal powers in the correlation output. The comparison of the efficiency of newly developed acquisition techniques against the software receiver conventional approach is based on a same criterion threshold which is 3dB above the statistical highest peak noise. The S/N used in the simulation input is the guaranteed minimum level required by the GPS standard. Since antenna patterns are different from receiver to receiver, an omnidirectional pattern is assumed. Finally the real collected data from our facility is used to run the different approaches and demonstrate the feasibility of each approach.

In our lab, the frequency of the received signal is 1575.42 MHz and is down converted to $f_0\approx1.25$MHz. An A/D converter samples the data at 5MHz to cover the base band of the C/A code. Therefore, this sampling rate is also used in the simulation for performance evaluation. However, the analysis and methodology discussed here are sampling rate independent and the generality is preserved. All the software effort is developed under MATLAB by MATHWORKS

CONVENTIONAL SOFTWARE RECEIVER APPROACH AND PERFORMANCE ANALYSIS

The conventional software receiver approach applies the DFT to the received signal to convert it from time domain into frequency domain.[1] The sampled received signal can be represented by

$$s(m\Delta t) = \sum_{n=1}^{k} a_n c_n(m\Delta t + \tau_n) \sin(2\pi f_n m\Delta t + \theta_n) + N(m\Delta t) \tag{1}$$

Where n is the satellite number, a_n is the amplitude, c_n is the C/A code, τ_n is the time delay, f_n is a carrier frequency, θ_n is the initial phase of the satellite n, N is the background noise, and k is the total number of the contributing satellites.

There are 21 locally generated signals associated with each satellite. These signals are represented by

$$r_j(m\Delta t) = c_j(m\Delta t) \exp(j2\pi f_k\, m\Delta t) \tag{2}$$

where $c_j(m\Delta t)$ is the sampled C/A code of satellite j

and the carrier frequencies, f_k, of these locally generated signals cover 1.25MHz ±10 kHz with 1KHz spacing. To acquire the beginning time of the C/A code of a targeted satellite, 21 locally generated signals are correlated with the received signal by DFT correlation. The DFT correlation method applies a DFT to both the received signal and the locally generated signal and then multiplies the DFT result of the signal with the complex conjugate of the DFT result of the locally generated signals. The complete correlation can be obtained by applying an inverse DFT (IDFT) to the multiplication result. Since the correlation time is 1ms it also functions as a filter of 1kHz bandwidth. In general, the DFT spectrums of these locally generated signals are pre-calculated and stored in the computer. After correlation, an exhaustive search is applied to the 21 correlation results for a global peak. The global peak is compared with the threshold which is pre-determined by the correlation noise floor statistics. If the peak is above the threshold, then detection has been accomplished and the beginning time of the C/A code and the received carrier frequency of the particular satellite can be found within the accuracy of 200ns and 1kHz.

The performance of this approach is evaluated by estimating the signal and the noise level after the correlation process and how much the signal will exceed the threshold. Since this approach is linear and the signal from each satellite and the background noise are statistically independent to each other, we can apply superposition to the signal and variance calculation. A set of 5000 points of 1 ms sampled data of a sin wave with 1.25 MHz carrier frequency modulated by a selected C/A code is generated to represent the ideal received signal. It is correlated with 5000 points of sampled data of a locally generated signal with the center frequency at 1.25Mhz. The power of the peak of the correlation output data set is 2500^2(67.96dB) which is the desired output signal, S_c, and the variance of the uncorrelated autocorrelation result is $\sigma_{ac}^2 = 4.021\times10^3$ (36.0dB). If the carrier frequency of the ideal received signal is not the same as the center frequency of the locally generated signal, there will be a loss and the worst case will be 3dB when the frequency is ±500 Hz off the center frequency. The same set of ideal received signal data is cross correlated with a different locally generated signal which has a different C/A code or a different carrier frequency at 1kHz or more away from 1.25Mhz. Since there are 5 or 4 samples in each chip the cross correlation result is not limited to three level -1, 63, -65.[2] The cross correlation results represent noise to the desired signal. Denote σ_{cc}^2 as their variance and it is also the averaged power of this noise set (DC for this output set is zero). The averaged value of σ_{cc}^2 from 9 cross correlation results is $4.44\times10^3\pm3.3\%$ (36.47dB). The variation of signal level from any satellite is 2.5dB when it moves from the horizon to the zenith. Therefore, the maximum of this noise is 7.895×10^3 (38.97dB). For an omnidirectional antenna, the typical minimum S/N behind the first low noise amplifier is -15dB.[3] A set of 5000 points of Gaussian distributed pseudo random numbers with variance of 16 is generated to represent the background noise (15 dB above the signal) and correlated with the 9 locally generated signals. Denote the variance (also averaged power) of the noise results as $\sigma_{nr}^2 + j\sigma_{ni}^2$. The average of σ_{nr}^2 and σ_{ni}^2 from 9 correlation results are $4.08\times10^4\pm1.5\%$ and $4.0768.051\times10^4\pm1.7\%$ respectively. They are almost the same so replace them with σ_n^2. The amplitude detector is formed by taking the absolute value of the complex correlation results. The averaged noise power at output of the amplitude detector is $2\sigma_n^2 = 8.159\times10^4\pm1.9\%$ (49dB). The probability density function of the real part and the imaginary part of these noise correlation results are normal. Therefore, the probability density function for the amplitude detector results is Rayleigh and cumulative probability function for it is exponential.[4]

$$f(x) = \frac{x}{\sigma_n^2}\exp(-\frac{x^2}{2\sigma_n^2})$$
$$F(x) = \int_x^{\infty} f(t)dt = \exp(-\frac{x^2}{2\sigma_n^2}) \tag{3}$$

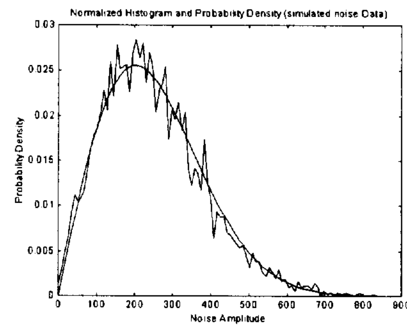

Figure 1: Normalized histogram and probability density function (simulated noise)

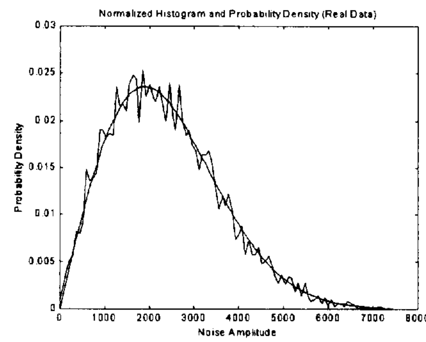

Figure 2: Normalized histogram and probability density function (field collected data)

As shown in Figure 1 and 2, the smooth curves are plotted with σ_n^2 and the zigzag curves are normalized histogram of the uncorrelated noise data. They match very well. Figure 1 uses the simulated noise and Figure 2 uses the field collected data. Comparing the noise from different noise sources, the background noise is dominant. For convenience, we only use the background noise in the noise threshold calculation. With 5000×21 points of output data (a very good sample size), the highest peak of these noise data sets will statistically be in the neighborhood of N_0 such that $F(N_0)= \exp(-N_0^2/2\sigma_n^2)=1/5000/21$. From this equation, N_0^2 is $23.12\sigma_n^2$ and N_0 is $4.8\sigma_n$. If the detection threshold is set 3 dB above N_0 then the threshold will be $6.79\sigma_n$. Now assume the signal is at worst case where the signal frequency is at the edge of the matched filter. The signal is 2.4 dB above the threshold.

Denote N=5000 as the number of points in 1ms. The computation of this approach includes 1 DFT for input data, 21N complex multiplications and 21 IDFTs for correlation. The time needed is approximately the same as that required for 23 DFTs. Following that, 21 N-point exhaustive search are needed to locate the global peak.

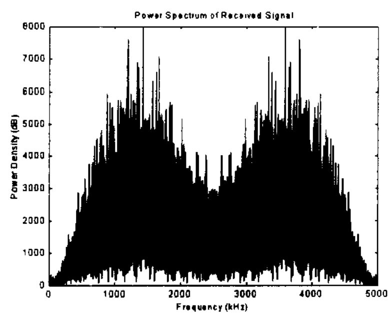

Figure 3: DFT spectrum of the received signal

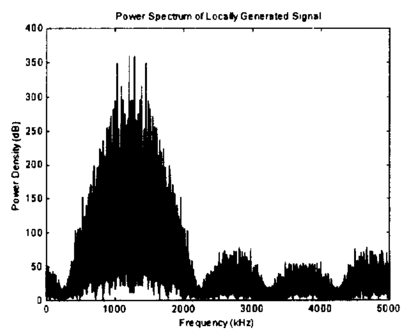

Figure 4: DFT spectrum of the locally generated signal

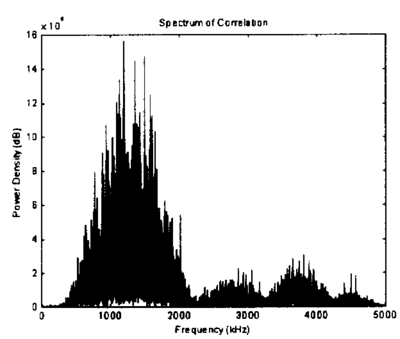

Figure 5: Spectrum of correlation (the multiplication of the spectrum of Figure 3 and the conjugate spectrum of Figure 4)

SINGLE SIDE BAND APPROACH AND PERFORMANCE ANALYSIS

The single side band approach is one of the new efficient approaches presented by this paper to carry out the conventional approach mentioned above. Since the Doppler variation is only ±10 kHz, figure 3 is a typical spectrum of 5000 samples of the received signal. Figure 4 is the spectrum of the 5000 samples of the locally generated signal. This signal is complex, and the spectrum is a single side band. Figure 5 is the result of multiplying the spectrum of the received signal by the complex conjugate of the spectrum of the locally generated signal. It is also a single side band. Compared to the power in the range from 1 to 2500 kHz, the power in the range of 2500 to 5000 kHz is very minimal and negligible. This approach takes advantage of this characteristic. Apply a 5000-point DFT to the received data, take the first 2500 points of the result, multiply them with the first 2500 points of 21 pre-calculated and stored complex conjugates of the 5000-point spectrums of the 21 locally generated signals, and perform a 2500-point IDFT to each of the 21 multiplication results. This is 2500-point DFT circular correlation. The 21 correlation results are exhaustively searched for a global peak. The peak is compared with the threshold. If it is higher than threshold, then detection has been achieved. Since the 2500-point inverse DFT is applied to the first half of the 5000-point DFT result, it is equivalent to re-sampling the time domain signal at half the original sampling rate. The time resolution is reduced by half but the S/N is almost unchanged. Since the S/N is barely degraded, the lost time resolution can be regained by the 3-point curve fitting using the peak signal and its' 2 neighbor signal points. The time of the peak will round to the time of the peak if 5000-point approach had been used. If 2048 instead of 2500 points is taken from the center of the first half of 5000-point spectrum for this approach, the efficiency can be further improved. It is because, as shown in Figure 5, the first 226 points and last 226 points of the first 2500 points of the spectrum are still very small. With this further reduction of points, a 2048-point FFT algorithm can be applied and the loss of S/N is very small. The re-sampling interval for this case is 5000/2048×200ns. Compared with the conventional approach, the efficiency improvement is obvious. Depending on the DFT algorithm used, it is definitely more than double. The performance evaluation procedures of this approach are the same as the conventional approach above. The simulation results from the same received signal and noise are listed in Tables 2 and 3 for the approach using 2500 points and 2048 points. Since the number of noise samples in a correlation output is 2500 or 2048, the highest noise peak will statistically be N_0 such that $\exp(-N_0^2/2\sigma_n^2)$ =

$1/2500/21$ or $\exp(-N_0^2/2\sigma_n^2) = 1/2048/21$. They are also listed in the tables.

The FFT and IFFT algorithm generally used is as follow.

$$X(m) = \sum_{n=1}^{N} x(n)\exp(\frac{-jnm2\pi}{N})$$
$$x(n) = \frac{1}{N}\sum_{m=1}^{N} X(m)\exp(\frac{jnm2\pi}{N}) \quad (4)$$

Table 1 Parameters for 5000 points (Conventional approach)

S_c	2500^2(67.96dB)
σ_{ac}^2	4.021×10^3 (36.0dB)
σ_{cc}^2	$4.44\times10^3\pm3.3\%$ (36.47dB)
$2\sigma_n^2$	$8.159\times10^4\pm1.9\%$(49dB).
N_O	$4.8\sigma_n$
V_T	6.79 σ_n (62.6dB)
S_c/V_T	5.4dB (2.4 dB if ±500Hz off the center)

Table 2 Parameters for 2500 Points

S_c	4175^2(72.4dB)
σ_{ac}^2	1.5532×10^4 (41.9dB)
σ_{cc}^2	1.7168×10^4 (42.35dB)±3.5%
$2\sigma_n^2$	3.01×10^5(54.79dB)±3.6%
N_O	$4.66\sigma_n$(65.13dB)
V_T	65.13+3=68.13dB
S_c/V_T	4.27dB (1.27 dB if ±500Hz off the center)

Table 3 Parameters for 2048 Points

S_c	5483^2(74.78dB)
σ_{ac}^2	2.312×10^4 (43.263dB)
σ_{cc}^2	2.546×10^4 (44.06dB)±3.5%
$2\sigma_n^2$	4.273×10^5(56.31dB)±2.07%
N_O	$4.62\sigma_n$(66.59dB)
V_T	66.59+3=69.59dB
S_c/V_T	5.19dB (2.19 dB if ±500Hz off the center)

Since the spectrum of the received signal and the pre-stored locally generated signal is generated by DFT equation 4 with N=5000 and the correlation result is based on IDFT equation 4 with N=2500 or 2048 As you may notice all the numbers in the Tables 2 and 3 are 4 and $(5000/2048)^2$=5.96 times higher. However, the S_c/V_T is not effected.

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SIGNAL DELAY-AND-MULTIPLY APPROACH AND PERFORMANCE ANALYSIS

Professor Martin Tomlinson of University of Plymouth, Devon, United Kingdom proposed an approach to perform acquisition on complex code division multiple access (CDMA) signals. The GPS signal is a CDMA signal. His approach is explained by using one satellite signal but more than one signal is used in the simulation. Let the complex input signal s(t) is

$$s(t) = A(t)e^{j2\pi f_0 t} \quad (5)$$

where A(t) is the amplitude and C/A code and f_0 is the frequency of the signal.

If this signal is delayed by time τ, the result is

$$s(t-\tau) = A(t-\tau)e^{j2\pi f_0(t-\tau)} \quad (6)$$

Now create a new signal by multiplying s(t) and the complex conjugate of s(t-τ), the result is

$$\begin{aligned} s_n(t) &= s(t)s(t-\tau)^* \\ &= A(t)A(t-\tau)e^{j2\pi f_0 t}e^{-j2\pi f_0(t-\tau)} \quad (7) \\ &= A(t)A(t-\tau)e^{j2\pi f_0\tau} \end{aligned}$$

This signal does not have any frequency component; therefore, one needs only to find the beginning of the C/A code. In order to find the beginning of the C/A code, the reference signal A(t)A(t-τ) is used to correlate with the new code which is generated by multiplying the C/A code of the satellite with a τ delayed version of itself. Once the beginning of C/A code is found, it is easy to find the frequency by performing one frequency search. Therefore, the approach does not require searching multiple frequency bins. Although this approach can save acquisition time, the multiplication operation increases the noise in the process.

The above approach can only be applied to complex signals. If the input signal is real, the real signal must be converted to a complex signal. The process of conversion is not only complicated, but it causes a transient effect. The GPS signal collected in our laboratory is real. It is relatively difficult and expensive to collect complex signals through an I/Q (In-phase and the Quadrature) detector or Hilbert Transformation. A more practical approach based on this concept is presented here. The input signal is

$$s(t) = A(t)\sin(2\pi f_0 t) \quad (8)$$

This signal is delayed by τ, and the result is

$$s(t-\tau) = A(t-\tau)\sin[2\pi f_0(t-\tau)] \quad (9)$$

The new signal can be created by multiplying s(t) and s(t-τ) as

$$s_n(t) = s(t)s(t-\tau)$$
$$= A(t)A(t-\tau)\sin(2\pi f_0 t)\sin[2\pi f_0(t-\tau)]$$
$$= \frac{A(t)A(t-\tau)}{2}\{\cos(2\pi f_0\tau) - \cos[4\pi f_0(t-\tau/2)]\} \qquad (10)$$

This equation contains two components, a dc term $\cos(2\pi f_0\tau)$ and a high frequency term $\cos[4\pi f_0(t-\tau/2)]$. The dc term can be used to find the beginning of the C/A code.

Strictly speaking, it is difficult to use this approach for acquisition. In order to make this approach function properly, one must make $\cos(2\pi f_0\tau)$ close to ± 1. Because the input frequency f_0 is unknown, it is very difficult to achieve this goal. However, for our GPS receiver application, it is easy to accomplish this goal. In receiving the signal, the input frequency 1575.42 MHz is down converted to $f_0 \approx 1.25$MHz. If one choose a delay time τ such that

$$\cos(2\pi f_0\tau) = \pm 1$$

i.e. $$f_0\tau = \pm\frac{n}{2} \quad where \quad n = 1, 2, 3, ... \qquad (11)$$

which implies

$$\tau = 0.4n\ \mu s$$

Now, let $\tau_0 = 0.4\mu s$ and also take the Doppler frequency shift of ±10KHz into consideration. The value of $f_0\tau$ for first several delay times are

$$f_0\tau_0 = \begin{cases}(1.25\times10^6 - 10\times10^3)\times0.4\times10^{-6} = .496\\(1.25\times10^6 + 10\times10^3)\times0.4\times10^{-6} = .504\end{cases}$$
$$2f_0\tau_0 = \begin{cases}.992\\1.008\end{cases} \quad 3f_0\tau_0 = \begin{cases}1.488\\1.512\end{cases} \qquad (12)$$

If n is small, the corresponding value of $\cos(2\pi f_0\tau)$ is very close to ± 1. Thus, this approach can work on real data. In order to find the beginning of the C/A code as mentioned before, the reference signal $A(t)A(t-\tau)$ is used to correlate with the newly generated signal. Once the beginning of the C/A code is determined, the frequency of the signal can be found easily from the frequency searching scheme.

This approach generates additional noise because of the multiplication operation. In order to improve the signal to noise ratio, several frames of continuous data and several correlations with different delay times are needed. The details will be discussed in the following paragraph. As in the above example, the correlation results from several delay times can be added together with the proper $\pm$ sign determined by $\cos(2\pi f_0\tau)$ to obtain the final result.

Without losing generality, assume the received signal includes two satellite signals with amplitude of one and a background noise with the variance of 16 (15dB above signal) for analysis of the performance of this approach. It has the same S/N in the input as conventional approach. After delay and multiplication, we can denote this result as

$$\begin{aligned}S(t) &= [a_1(t)+a_2(t)+4N(t)]\times\\&\quad[a_1(t-\tau)+a_2(t-\tau)+4N(t-\tau)]\\&= a_1(t)a_1(t-\tau)+a_2(t)a_2(t-\tau)\\&\quad+16N(t)N(t-\tau)+a_1(t)a_2(t-\tau)\\&\quad+a_2(t)a_1(t-\tau)+4N(t)a_1(t-\tau)\\&\quad+4N(t)a_2(t-\tau)+4N(t-\tau)a_1(t)\\&\quad+4N(t-\tau)a_2(t)\end{aligned} \qquad (13)$$

Where,

$$a_1(t) = A_1(t)\sin(2\pi f_1 t + \theta_1)$$
$$a_2(t) = A_2(t)\sin(2\pi f_2 t + \theta_2)$$

$A_1(t)$ and $A_2(t)$ are C/A code and the amplitude is one. The variance of N(t) is one.

In the analysis program, 5 sets of 5000 points of continuous sampling data are used to perform delay and multiplication. Currently the delay used is 10 sampling periods, or $\tau=2\mu s$. After that, 25000 points of the result is added frame by frame to form one frame of 5000 data points. Since this addition is a coherent add for signals and non-coherent add for noises. The result will improve the S/N by 7dB. This frame of 5000 data points is correlated with a selected 'new C/A code' which is locally generated by the multiplication of the C/A code of the selected satellite with the delayed version of same code. The delay time is also $\tau=2\mu s$. Since there is no carrier frequency involved only one DFT correlation is needed. The peak of this output could be located by only one search in this correlation result if the signal were above the threshold. The whole process other than the delay-and -multiply operation in the beginning is linear and all terms in equation 13 is statistically independent, therefore, superposition can be applied to the signal and variance in this analysis. In this analysis, 5 continuous frames of pure delay-and-multiply signal, $a_1(t)a_1(t-\tau)$, is generated, frame by frame added, and correlated with it's associated 'new C/A code'. The peak of the correlation result is the desired output signal. The power of the peak is $S_c = 12500^2 = 81.94$dB and the variance of the rest, σ_{ac}^2, is 49.1dB. The same set of data is correlated with 9 'new C/A codes' with the same delay but associated with 9 other satellites. The cross correlation variance σ_{cc1}^2 is $9.98\times10^4 \pm 5\%$(50dB). This is first kind of cross correlation noise. Five frames of data representing $a_1(t)a_2(t-\tau)$ are also generated, frame by frame added, and correlated with the same 9 'new codes' as mentioned. The averaged $\sigma_{cc2}{}^2$ is $3.31\times10^3\pm5\%$(35dB). This is the second kind of cross correlation noise. The same process is applied to $4a_1(t)N(t-\tau)$ to get $\sigma_{ns}{}^2 = 2.0855\times10^5\pm3.28\%$ (53.2dB), and applied to $16N(t)N(t-\tau)$ to get the averaged $\sigma_{nn}{}^2 = 6.3632\times10^6\pm2.5\%$ (68dB). Comparing all the noise sources, the background noise (represented by $\sigma_{nn}{}^2$) is still dominant. It is still reasonable to use this noise alone for performance evaluation. The PDF of the noise is a normal distribution and the cumulative probability function is the error function. The results by using actual data and simulated noise are shown in figures 6 and 7.

$$f(|x|) = \frac{2}{\sigma_{nn}\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_{nn}{}^2}}$$
$$F(|x|) = 1 - \int_0^{|x|} \frac{2}{\sigma_{nn}\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_{nn}{}^2}} dt \qquad (14)$$

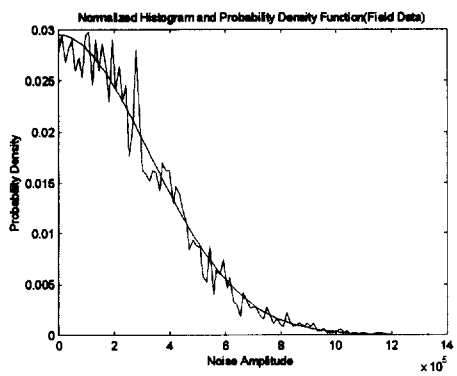

Figure 6: Normalized histogram and probability density function (field collected data)

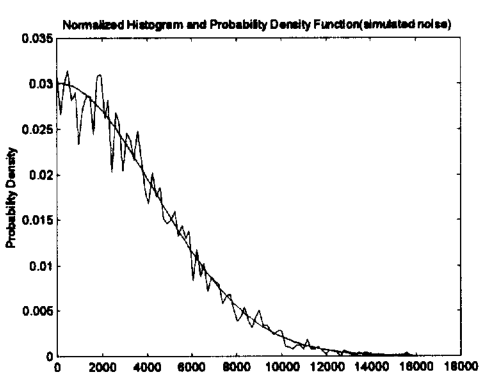

Figure 7: Normalized histogram and probability density function (simulated noise)

The highest peak noise statistically should be N_0 such that $F(|N_0|)=1/5000$. Solve this equation and get $|N_0| = 2.63(2)^{1/2}\sigma_{nn} = 3.72\sigma_{nn}$=79.41dB. If the threshold ,V_T, is 3 dB above this value, V_T will be 82.41 dB. The signal is below the threshold. For the same or little bit better performance two more complete correlation processes are needed for addition (with proper sign). Because the three correlation results are totally independent to each other. The final variance of the noise is $3\sigma_{nn}{}^2 = 1.909\times10^7\pm2.5\%$(72.81dB). The statistical highest peak noise $|N_0| = 2.63(6)^{1/2}\sigma_{nn} = 6.44\sigma_{nn}$=8 4.21dB. If threshold ,V_T, is 3 dB above this value, V_T will be 87.21 dB. After the three correlation summations, the desired signal is coherently added. Therefore, the amplitude is increased 3 times (9.54dB) and the final desired signal is 91.48dB, which is 4.27dB above the threshold.

Denote N as the number of points. The calculation for this approach using 5 continuous block of data and 3 correlations includes 15N real multiplications and 15N real additions plus 3 DFTs for input data, 3N complex multiplications and 3 IDFTs for correlation, and N multiplications and 1 DFT for frequency resolution of 1Khz. Roughly, 8 DFT computation times are needed compared with 23 for the conventional approach. One N-point search time is required compared with 21 N-point search times for the conventional approach. In practice, if more S/N is needed for a weak signal, more than 5 blocks of continuous data can be used for the delay-and-multiply process for each delay time, and more than 3 correlations with different delay times can be added. In general, if M is the number of blocks of continuous data in the process and L is the number of the correlation additions, the S/N improvement is 10logML.

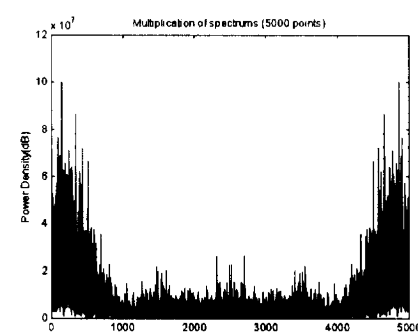

Figure 8: DFT of correlation (5000 points delay-and-multiply approach)

Figure 8 is the typical correlation spectrum of the delay-and-multiply approach. It is one stage before the IDFT. The energy contained in the frequency range of 1024 to 3975 kHz is very small. Delete the spectrum in [1024, 3975] and move the spectrum in [3976, 5000] to [1024, 2048] to form the 2048 point spectrum. If the spectrum of all the 'new codes' and the summing frame can be constructed in the same way, then 2048 point single side band technique mentioned above can also be used here to reduce the processing time. From the simulation of one correlation, the desired signal, S_c, is 26598^2 =88.5dB. After summation of 3 correlation results, the final variance of the noise is $3\sigma_{nn}^2=1.014\times10^8$. The statistical highest peak noise is $|N_0|$ such that $F(|N_0|)=1/2048$. Sovle this equation yields $|N_0| = 2.466(6)^{1/2}\sigma_{nn} = 6.04\sigma_{nn}$ =90.9dB. The threshold, V_T, is 93.9 dB. The amplitude of desired signal is increased 3 times (9.54dB) and the final desired signal is 98.04dB, which is 4.14dB above the threshold. The performance almost remains the same. Since the 2048 point operation can apply only to the spectrum multiplication and IDFT stages in this approach, the amount of savings is not as significant as with the conventional approach. The result is demonstrated in the following section.

RESULTS FROM ACTUAL COLLECTED DATA

The signals of satellite 6 and 17 are two stronger signals in our field data. They are selected for demonstration because the results are easier to observe. Figures 9-16 are the results of four approaches applied to a set of field collected data. Figures 9-12 are the results for satellite 6 and Figures 13-16 are the results for satellite 17. The four approaches used are the conventional 5000 points approach, the 2048 single side band approach, the 5000 point delay-and-multiply approach, and 2048 point delay-and-multiply single side band approach. The delay-and-multiply approach used 5 blocks of continuous data and 3 correlation with time delays of τ=2μs, τ=4μs, and τ=6μs. By observation, all the desired signals are about 15dB above the noise floor for all the approaches. Their performances are very close to what the previous performance analysis predicts.

Figures 2 and 3 are the results of field collected data cross correlated with a locally generated signal associated with an arbitary satellite. The variations of the variances of the cross correlation for both the conventional approach and the delay-and-multiply approach are very small (the standard deviation is less than 5% with one hundred of simulation runs). If the variance of the received signal can be measured before the acquisition process, then PDFs and cumulative probability functions of the correlation noise and the signal plus noise can be defined and the threshold can be set based on a proper criterion. Instead of searching for a global peak for detection, the threshold can be used to compare against each data point of the correlation results immediately after each correlation process. Whenever the data point is above the threshold, detection is achieved and the acquisition process can be stopped. On average, this further reduces the acquisition by half.

CONCLUSION

The methodology used for variance in this paper can be used to measure the variance of the correlation noise floor. Based on this variance, a practical threshold can be developed. The delay-and-multiply approach is not very practical for a weak signal but if a GPS receiver is only interested in a few strong satellite signals, then this approach can be used very efficiently. Since the efficiency of the single side band technique is very obvious and the S/N loss is so trivial, it is highly recommended this technique be applied to the conventional approach and to the multiply and delay approach.

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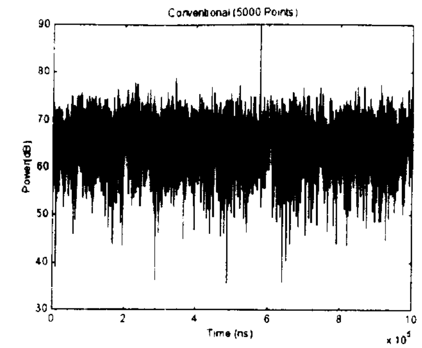

Figure 9: Conventional approach for satellite 6

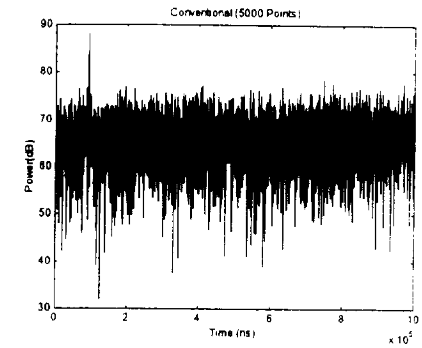

Figure 13: Conventional approach for sattelite 17

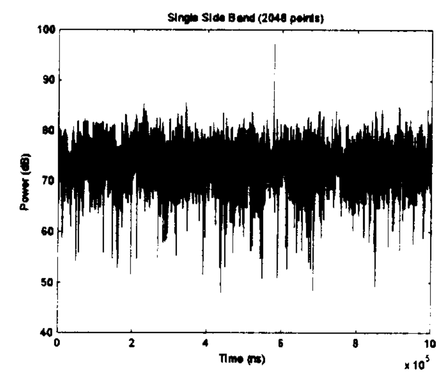

Figure 10: 2048 points single side band approach for satellite 6

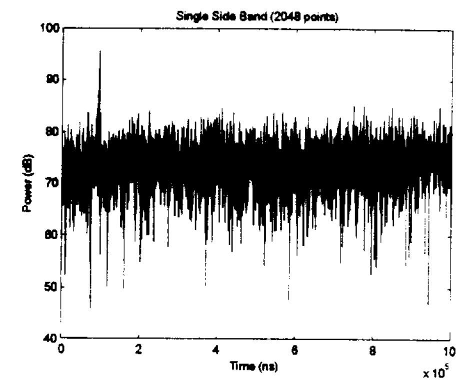

Figure 14: 2048 points single side band approach for satellite 17

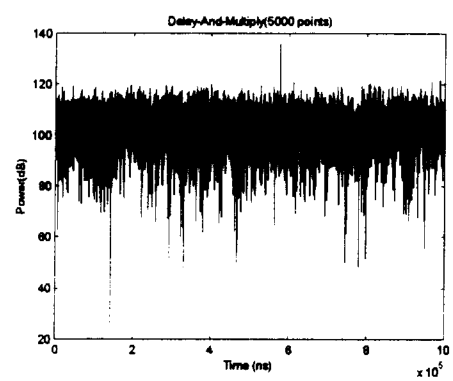

Figure 11: Delay-and-multiply approach for satellite 6

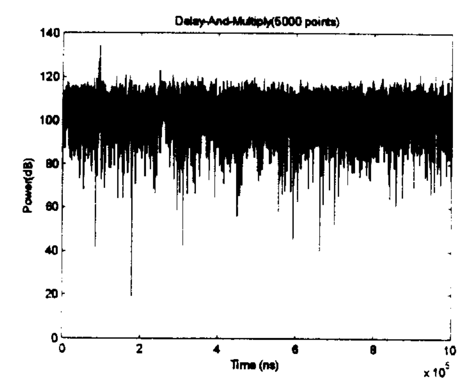

Figure 15: Delay-and-multiply approach for satellite 17

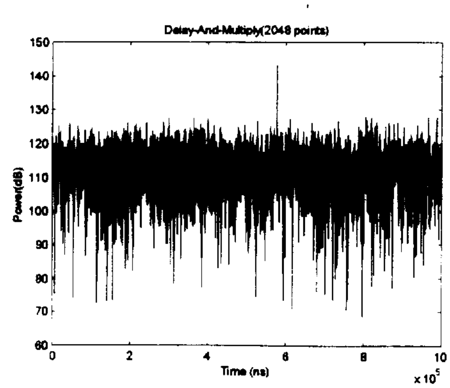

Figure 12: Delay-and-multiply 2048 single side band approach for satellite 6

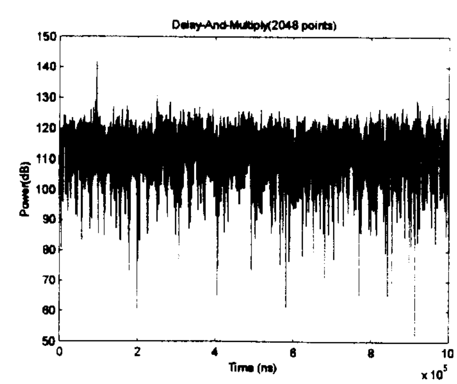

Figure 16: Delay-and-multiply 2048 single side band approach for satellite 17