A New Fast Acquisition Algorithm for GPS Receivers

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Abstract: In many GPS applications, such as high dynamics vehicles or an appliance which uses sleep mode to save power consumption, a fast acquisition of signal is strongly required. In this paper, a new fast method to acquire GPS signal is proposed. The proposed methods find the candidates of Doppler shift using Squared-D algorithm, which is based upon a signal squaring method and dramatically reduces the search space of Doppler shift. This paper also proposes XMC (eXtended Multi-Correlator) to efficiently search C/A code. It is distinguished from the multi-correlator in that the combined replica of several codes is used. The expected performance of XMC method is equal to that of the multi-correlator while computational burden is reduced to 90%.

Keywords: **fast acquisition, XMC, extended multi-correlator, Squared-D**

1. Introduction

Most GPS receivers perform time-consuming two-dimensional search to acquire the signal from satellites because they have no code phase and Doppler shift information. It takes at least a few seconds even if the almanac is known. But in many applications, such as high dynamics vehicles or an appliance which uses sleep mode to save power consumption, a fast acquisition of signal is strongly required.

Correlators for fast acquiring the GPS signal are typically categorized into three types: multi-correlator, matched filter, and FFT[7]. However, these methods have limited applications because of computational burdens, power consumptions and their complexities.

A new algorithm to acquire GPS signal is proposed in this paper. It finds the candidates of Doppler shift using a new method called Squared-D, which is based upon a signal squaring method. It dramatically reduces the search space of Doppler shift because the Squared-D method gives several numbers of candidates of Doppler shift. Another new method called XMC (eXtended Multi-Correlator) is proposed to efficiently search C/A code. XMC is an extension of the multi-correlator method. It is distinguished from other method in that the combined replica of several codes is used instead of separate ones. The phase of code can be estimated with small numbers of correlator's arms under this structure. Theoretically, 90% numbers of arms are used to get an equivalent performance of the multi-correlator. Thus the expected performance of XMC method is equal to that of the multi-correlator while the complexities and computational burdens are reduced to 90%. It is expected that the proposed method be easily adapted to many applications where the fast acquisition are required.

2. Acquisition Algorithms

2.1 The Serial and Parallel Acquisition

GPS signal acquisition is a search process. The C/A code dimension is associated with the replica code. The Doppler dimension is associated with the replica carrier. The uncertainty in both Doppler and C/A code phase suggests that a 2-dimensional uncertainty region must be searched in order to locate the received signal. Figure 1 illustrates the 2-dimensional search region. When the GPS receiver has no almanac available (called by cold start), the code uncertainty region is 1023 chips, and the Doppler uncertainty region is about −11000Hz ~ +11000Hz typically.

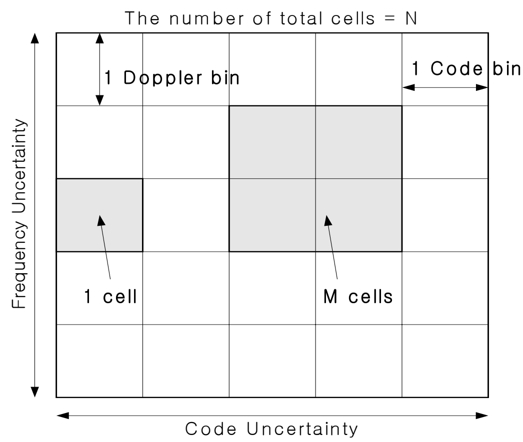

Figure 1. Code / Doppler uncertainty region

In Figure 1, each Doppler search increment is called a Doppler bin, and each C/A code phase search increment is called a code bin. The combination of one code bin and one Doppler bin is a cell. In a serial search, the searching procedure is executed one cell at a time in sequence, while in a parallel search (Multi-correlator), groups of cells (M cells) are tested simultaneously. Typically, the code bin size is 1/2 chip of CA code [1]. The Doppler bin size is about 2/(3T), where T is the PDI (pre-detection integration) time [1].

A block diagram of a single dwell acquisition system is shown in Figure 2. The input signal is the combination of the satellite signal (s(t)) and the noise (n(t)). This input signal is mixed with the generated carrier and code, and passed through an integrated-and-dump (IDF) with a pre-detection integration time of T. For a fixed dwell scheme, a large number, N_B, of squared-law-detected samples of the coherent integration are accumulated, summed, and compared with a preset threshold.

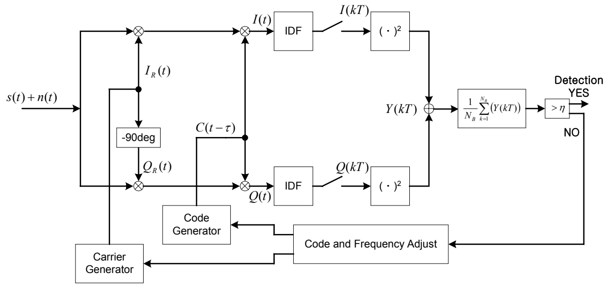

Figure 2. A single dwell C/A acquisition system

In Figure 2, the input signal (the received SV signal and the noise) can be written as [4]

$$s(t) = A \cdot C(t) \cdot \cos(\omega t + \phi(t)) + n(t) \tag{1}$$

where A is the received SV signal amplitude, $C(t)$ is the C/A code modulation, ω is the IF carrier frequency, $n(t)$ is the noise, and $\phi(t)$ is the carrier phase which includes the 50 bps bi-phase data modulation. The generated in-phase and quadrature mixing signals are written as

$$\begin{aligned} I_R(t) &= 2 \cdot \cos(\hat{\omega} t + \hat{\phi}) \\ Q_R(t) &= -2 \cdot \sin(\hat{\omega} t + \hat{\phi}) \end{aligned} \tag{2}$$

where $\hat{\omega}$ is an estimate of the received frequency and $\hat{\phi}$ is an arbitrary phase. The demodulated in-phase and quadrature phase signals are mixed with the generated C/A code ($C(t-\tau)$) which is for despreading the spread spectrum signal. The despreaded signals are then filtered by IDF. The filtered signals are written as [4]

$$\begin{aligned} I(T) &= \frac{1}{T}\int_0^T s(t) \cdot I_R(t) \cdot C(t-\tau)dt + \frac{1}{T}\int_0^T n(t) \cdot I_R(t) \cdot C(t-\tau)dt \\ &\quad \left(\frac{1}{T}\int_0^T n(t) \cdot I_R(t) \cdot C(t-\tau)dt \equiv n_i(T) \right) \\ &= \frac{2A}{T} \cdot R(\tau) \cdot \int_0^T \cos(\omega t + \phi(t)) \cdot \cos(\hat{\omega} t + \hat{\phi})dt + n_i(T) \\ &\quad (\mathit{the\ high\ frequency\ term}\ (\omega(t) + \hat{\omega})\ \mathit{to\ zero\ in\ IDF}) \\ &\cong A \cdot \frac{\sin(\omega_e \cdot T/2)}{(\omega_e \cdot T/2)} \cdot R(\tau) \cdot \cos(\omega_e \cdot T/2 + \phi_e) + n_i(T) \end{aligned} \tag{3}$$

$$Q(T) = A \cdot \frac{\sin(\omega_e \cdot T/2)}{(\omega_e \cdot T/2)} \cdot R(\tau) \cdot \sin(\omega_e \cdot T/2 + \phi_e) + n_q(T) \tag{4}$$

where $\omega_e = \omega - \hat{\omega}$, $\phi_e = \phi(t) - \hat{\phi}$, and $n_i(T)$ is the in-phase noise, $n_q(T)$ is the quadrature phase noise. $n_i(T)$ and $n_q(T)$ are zero-mean Gaussian statistically independent random variables with average noise power given by $\sigma^2 = N/T$, N is the single side noise power spectral density, and T is the PDI (Predetection Integration) time. The correlation function $R(\tau)$ between the incoming C/A code and the generated C/A code is given by

$$R(\tau) = \begin{cases} 1 - |\tau| & ;\ \tau < 1 \\ 0 & ;\ \mathit{otherwise} \end{cases} \tag{5}$$

When the received SV signal and generated codes are exactly synchronized, the pre-detection integration power without noises is

$$\begin{aligned} Y(T) &= I^2(T) + Q^2(T) \\ &= A^2 \left(\frac{\sin(\omega_e \cdot T/2)}{\omega_e \cdot T/2} \right)^2 \end{aligned} \tag{6}$$

The test statistic for signal acquisition is given by [4]

$$Z(kT) = \frac{1}{N_B} \sum_{k=1}^{N_B} \left[I^2(kT) + Q^2(kT) \right] \tag{7}$$

where N_B is the number of post-detection samples. The random variable, $Z(kT)$, is the output of the square-law detector. A threshold value, η, is established for desired detection and false alarm probabilities. If $Z(kT)$ exceeds η in any given cell(s), a verification procedure is initiated; otherwise, noise only is assumed, and the procedure is repeated for the next cell [3].

The probability of the false alarm, P_{FA}, is the probability that Z exceeds the threshold η when only the noise is present. We can write the probability of the false alarm as

$$P_{FA} = \Pr\{Z > \eta\} = \int_\eta^\infty P_n(Z)dZ \tag{8}$$

The detection probability P_D is the probability that Z exceeds the threshold η when the signal and noise are present. The detection probability can be written as

$$P_D = \Pr\{Z > \eta\} = \int_\eta^\infty P_{s+n}(Z)dZ \tag{9}$$

Let's normalize the random variable Z and the threshold η by $2\sigma^2 / N_B$ as

$$\begin{aligned} Z^* &\equiv \frac{Z \cdot N_B}{2 \cdot \sigma^2} \\ \eta^* &\equiv \frac{\eta \cdot N_B}{2 \cdot \sigma^2} \end{aligned} \tag{10}$$

Since, by the assumption, the $Y(kt)$'s are independent random variables, for large N_B (N_B>10), Z^* is approximately Gaussian distributed. Thus, the probability of the false alarm and the detection probability are [6]

$$\begin{aligned} P_{FA} &\cong \int_{\eta^*}^\infty \frac{1}{\sqrt{2 \cdot \pi \cdot N_B}} e^{-\frac{(Z^* - N_B)^2}{2 \cdot N_B}} dZ^* \\ &= Q\left(\frac{\eta^* - N_B}{\sqrt{N_B}} \right) \\ &\equiv Q(\beta) \end{aligned} \tag{11}$$

$$\begin{aligned} P_D &\cong \int_{\eta^*}^\infty \frac{1}{\sqrt{2 \cdot \pi \cdot N_B \cdot (1 + 2 \cdot \gamma)}} e^{-\frac{(Z^* - N_B \cdot (1+\gamma))^2}{2 \cdot N_B \cdot (1+2\gamma)}} dZ^* \\ &= Q\left(\frac{\beta - \sqrt{N_B} \cdot \gamma}{\sqrt{1 + 2 \cdot \gamma}} \right) \end{aligned} \tag{12}$$

where $\gamma = A^2 / (2 \cdot \sigma^2)$ and $Q(x)$ is the Gaussian probability integral.

A good approximation to the required N_B as a function of P_{FA} and P_D is [3].

$$N_B(P_{FA}, P_D) \cong \frac{\pi}{2} \cdot \left(\frac{\ln^{0.5}\left(\frac{1}{4 \cdot P_{FA}}\right) + \ln^{0.5}\left(\frac{1}{4 \cdot P_D \cdot (1 - P_D)}\right) \cdot \sqrt{1 + 2 \cdot \gamma}}{\gamma} \right)^2 \quad (13)$$

The mean acquisition time for serial search may be expressed as [3]

$$\overline{T}_{Acq} = \left(\frac{2 - P_D}{2 \cdot P_D}\right) \cdot \left(k_P \cdot P_{FA} + 1\right) \cdot N \cdot T \cdot N_B\left(P_{FA}, P_D\right) \quad (14)$$

where k_p is defined as a multiplier of the dwell period, and N is the number of total cells.

In the case of a parallel search (in the multi-correlator), several cells (M) are tested concurrently for signal presence. The mean acquisition time for parallel search may be written as [3]

$$\overline{T}_{Acq} = \left(\frac{2 - P_D}{2 \cdot P_D}\right) \cdot \frac{k_P \cdot \left(1 - \left(1 - P_{FA}\right)^M\right) + 1}{M} \cdot N \cdot T \cdot N_B\left(P_{FA}, P_D\right) \quad (15)$$

2.2 XMC (eXtended Multiple Correlator)

There are many techniques for the fast acquisition of GPS signal. Among them, multi-correlator, matched filter and FFT are most popular. The multi-correlator uses a larger number of correlation arms, and correlation values between the received code and the replica code generated at each arm are obtained simultaneously. In matched filter, mathematically equivalent form of the multi-correlator, correlations are performed between the replica code and a set of the received codes that are delayed at each arm. FFT is a frequency domain equivalent form of the matched filter or the multi-correlator. It reduces processing time significantly and can be used in tracking process.

This paper proposes a new fast method to search C/A code called XMC (eXtended Multi-Correlator). In XMC, the combined replica of several codes is used instead of separate ones. A block diagram of XMC correlation arm is shown in Figure 3.

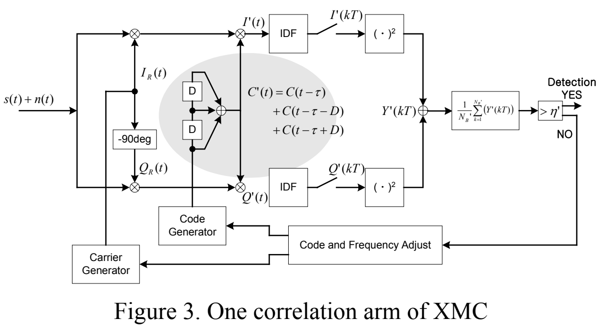

Figure 3. One correlation arm of XMC

The main feature of this correlation technique is shown in the shadowed block of Figure 3. Differently from other correlation techniques, the demodulated in-phase and quadrature phase signals are mixed with the combined replica of three C/A codes for despreading the spread spectrum signal. Other structures are same as the single/multiple correlator.

The combined replica of three C/A codes is written as

$$C'(t) = C(t - \tau) + C(t - \tau - D) + C(t - \tau + D) \quad (16)$$

where D is the desired value. Thus, the IDF output signals are written as

$$I'(T) = \frac{1}{T} \cdot \int_0^T s(t) \cdot I_R(t) \cdot C'(t)dt + \frac{1}{T} \cdot \int_0^T n(t) \cdot I_R(t) \cdot C'(t)dt \quad (17)$$

$$Q'(T) = \frac{1}{T} \cdot \int_0^T s(t) \cdot Q_R(t) \cdot C'(t)dt + \frac{1}{T} \cdot \int_0^T n(t) \cdot Q_R(t) \cdot C'(t)dt \quad (18)$$

We define new variables

$$\begin{aligned} &\frac{1}{T} \cdot \int_0^T n(t) \cdot I_R(t) \cdot C'(t)dt \equiv n_i'(T) \\ &\frac{1}{T} \cdot \int_0^T n(t) \cdot Q_R(t) \cdot C'(t)dt \equiv n_q'(T) \end{aligned} \quad (19)$$

then the IDF output signals can be written as

$$\begin{aligned} \therefore I'(T) \cong A \cdot &\frac{\sin(\omega_e \cdot T/2)}{(\omega_e \cdot T/2)} \cdot \left[R(\tau) + R(\tau - D) + R(\tau + D)\right] \cdot \cos(\omega_e \cdot T/2 + \phi_e) \\ &+ n_i'(T) \end{aligned} \quad (20)$$

$$\begin{aligned} \therefore Q'(T) \cong A \cdot &\frac{\sin(\omega_e \cdot T/2)}{(\omega_e \cdot T/2)} \cdot \left[R(\tau) + R(\tau - D) + R(\tau + D)\right] \cdot \sin(\omega_e \cdot T/2 + \phi_e) \\ &+ n_q'(T) \end{aligned} \quad (21)$$

When the received SV signal and generated codes are exactly synchronized (τ = 0), the pre-detection integration power without noises is given by

$$\begin{aligned} Y'(T) &= I'^2(T) + Q'^2(T) \\ &= A^2 \left(\frac{\sin(\omega_e \cdot T/2)}{\omega_e \cdot T/2}\right)^2 \cdot \left[1 + 2 \cdot R(D)\right]^2 \end{aligned} \quad (22)$$

Comparing Equation (21) with Equation (6), we can find that the pre-detection integration power in XMC is stronger than that in conventional (single/multiple) correlator. If D is a half chip (about 0.5 msec), then this value is quadruple as large as the value in Equation (6). The in-phase noise $n_i'(T)$ and the quadrature phase noise $n_q'(T)$ are zero-mean Gaussian statistically independent random variables. Therefore, the average power of each noise can be shown as

$$\begin{aligned} \sigma'^2 &= \left(3 + 2 \cdot R(-D) + 2 \cdot R(D) + 2 \cdot R(2 \cdot D)\right) \cdot \sigma^2 \\ &= \left(3 + 4 \cdot R(D) + 2 \cdot R(2 \cdot D)\right) \cdot N/T \end{aligned} \quad (23)$$

The probability of the false alarm, the detection probability and the mean acquisition time are same as Equation (8)~(15).

In XMC, several code phases can be tested in just one correlator arm. Therefore, the number of total code bins is decreased in XMC. We found that the number of total code bins is about 71.5 % of that of conventional (single/multiple) correlator through experiments results. In this case, theoretically, 90% numbers of arms are used to get an equivalent performance of the multi-correlator when the required probability of the false alarm is 10^{-4} and the required detection probability is 0.95. This value can be verified using Equation (11)~(15), (22)~(23).

2.3 Squared-D Searching Method

In practice, all kinds of GPS correlators must detect the SV signals in the carrier-phase (or Doppler) dimension by replicating the carrier frequency plus Doppler. Many candidates of Doppler must be searched when the GPS receiver has no almanac available. Otherwise, if GPS correlators can reduce the number of Doppler bin, these correlators can acquire the GPS signal faster than any other correlator.

This paper proposes a new method for reducing the number of Doppler bin. This new method is called by Squared-D searching method, and is based upon a signal squaring method for L2 codeless tracking [2]. A block diagram of Squared-D searching method is shown in Figure 4.

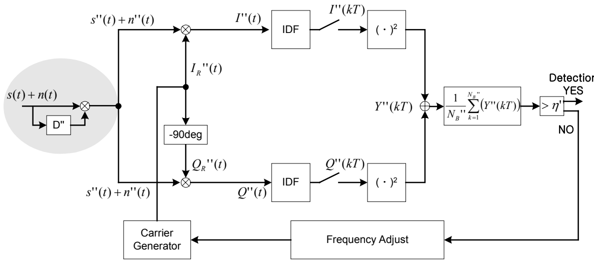

Figure 4. Squared-D searching method

The main feature of Squared-D searching method is shown in the shadowed block of Figure 4. This method is to square the L1 signal, that is, to multiply it by delayed itself, to remove the bi-phase C/A code modulation and result in a demodulated. Therefore, this correlator can't detect the SV number because the C/A code is removed, but can find twice of Doppler frequency. Details are following.

The squared of input signal is written as

$$\begin{aligned} s''(t)+n''(t) &= \left(s(t)+n(t)\right)\cdot\left(s(t+D'')+n(t+D'')\right) \\ &= \left(A\cdot C(t)\cdot\cos(\omega t+\phi(t))+n(t)\right) \\ &\quad\times\left(A\cdot C(t+D'')\cdot\cos(\omega(t+D'')+\phi(t+D''))+n(t+D'')\right) \end{aligned} \tag{24}$$

where D" is the desired value, but must be less than the reciprocal of IF carrier frequency. The generated in-phase and quadrature mixing signals are written as

$$\begin{aligned} I_R''(t) &= 2\cdot\cos(2\hat{\omega}t+\hat{\phi}) \\ Q_R''(t) &= -2\cdot\sin(2\hat{\omega}t+\hat{\phi}) \end{aligned} \tag{25}$$

Comparing Equation (25) with Equation (2), we can find the estimate of received frequency is twice of that of Equation (2). It means that the correlator detect twice of Doppler frequency rather than the Doppler frequency itself. Thus, the IDF output signals are written as

$$\begin{aligned} I''(T) &= \frac{1}{T}\cdot\int_0^T s''(t)\cdot I_R''(t)dt \\ &= \frac{2}{T}\cdot\int_0^T \Big(A^2\cdot C(t)\cdot C(t+D'')\cdot\cos(\omega t+\phi(t))\cdot\cos(\omega(t+D'')+\phi(t+D''))\cdot\cos(2\hat{\omega}t+\hat{\phi}) \\ &\quad + A\cdot C(t)\cdot\cos(\omega t+\phi(t))\cdot n(t+D'')\cdot\cos(2\hat{\omega}t+\hat{\phi}) \\ &\quad + A\cdot C(t+D'')\cdot\cos(\omega(t+D'')+\phi(t+D''))\cdot n(t)\cdot\cos(2\hat{\omega}t+\hat{\phi}) \\ &\quad + n(t)\cdot n(t+D'')\cdot\cos(2\hat{\omega}t+\hat{\phi})\Big)dt \\ &\cong \frac{2}{T}\cdot\int_0^T \Big(A^2\cdot C(t)\cdot C(t+D'')\cdot\cos(\omega t+\phi(t))\cdot\cos(\omega(t+D'')+\phi(t+D''))\cdot\cos(2\hat{\omega}t+\hat{\phi})\Big) \\ &\quad + n_i''(T) \\ &\cong \frac{A^2}{2}\cdot\frac{\sin(\omega_e''\cdot T/2)}{(\omega_e''\cdot T/2)}\cdot\cos(\omega_e''\cdot T/2+\phi_e'')\cdot R(D'')+n_i''(T) \end{aligned} \tag{26}$$

$$\begin{aligned} Q''(T) &= \frac{1}{T}\cdot\int_0^T s''(t)\cdot Q_R''(t) \\ &\cong \frac{A^2}{2}\cdot\frac{\sin(\omega_e''\cdot T/2)}{(\omega_e''\cdot T/2)}\cdot\sin(\omega_e''\cdot T/2+\phi_e'')\cdot R(D'')+n_q''(T) \end{aligned} \tag{27}$$

where $\omega_e''\equiv 2\cdot(\omega-\hat{\omega})$ and $\phi_e''\equiv\phi(t)+\phi(t+D)-\hat{\phi}$. Therefore, the pre-detection integration power without noises is given by

$$\begin{aligned} Y''(T) &= I''^2(T)+Q''^2(T) \\ &= \frac{A^4}{4}\left(\frac{\sin(\omega_e''\cdot T/2)}{\omega_e''\cdot T/2}\right)^2\cdot R(D'')^2 \end{aligned} \tag{28}$$

Comparing Equation (21) with Equation (28), we can find that the pre-detection integration power in Squared-D searching method is weaker than that in conventional (single/multiple) correlator. If D'' is 25 nsec and A is unity, then this value is about 24% of Equation (6). The in-phase noise $n_i''(T)$ and the quadrature phase noise $n_q''(T)$ are zero-mean Gaussian statistically independent random variables. Therefore, the average power of each noise can be shown as

$$\sigma''^2 = N^2/T, \tag{29}$$

The probability of the false alarm, the detection probability and the mean acquisition time are same as Equation (8)~(15).

For weak signal power and strong noise power, Squared-D searching method needs the longer PDI time. When GP2010 is used in RF/IF chipset and D'' is 175 nsec, PID time should be more than about 21 msec for acquiring Doppler frequency. If GP2021 used in RF/IF chipset, the Squared-D correlator has 12 channels, 8 visible satellites are presented, and the Doppler uncertainty region is about –11000Hz ~ +11000Hz, then it takes about 3.2 sec for finding Doppler candidates. Therefore, the number of Doppler bin is reduced to 8, and the GPS receiver using this Squared-D searching method can acquire the GPS signal dramatically faster than any other receiver.

3. Conclusions

This paper proposed a new algorithm to acquire GPS signal, Squared-D searching method and XMC. Squared-D searching method dramatically reduces the number of Doppler bins. XMC is distinguished from other multi-correlator in that the combined replica of three codes is used instead of separate ones, and 90% numbers of arms are used to get an equivalent performance of the multi-correlator.

Table 1. Acquisition performances

	TTFF [sec] (Min / Max)
Serial Search Correlator	30 / 120
Multi-Correlator	30 / 44
XMC	30 / 42
Serial Search Correlator with Squared-D searching method	33 / 49
XMC with Squared-D searching method	33 / 35

Table 1 shows the acquisition performance (TTFF, Time To First Fix) in cold start mode. This performance is calculated when each correlator has 12 channels, 8 visible satellites are presented, the Doppler uncertainty region is about –11000Hz ~ +11000Hz, serial search correlator has 3 correlator arms, and the multi-correlator and XMC have 20 correlator arms. In Table 1, we can see that XMC with Squared-D searching method has a much improved acquisition performance. It is expected that proposed methods be useful for GPS receivers in many applications, such as high dynamics vehicles or an appliance which uses sleep mode to save power consumption, a fast acquisition of signal is strongly required.

References

[1] E. D. Kaplan, *Understanding GPS : Principles and Applications*, Artech House, MA, 1996.

[2] J. Ashjaee, et. al., “Global Positioning System Receiver with Improved Radio Frequency and Digital Processing,” U.S. Patent No. 4,928,106, May 1990.

[3] J. B. Lozow, “Analysis for Direct P(Y)-Code Acquisition,” *NAVIGATION, Journal of The Institute of Navigation*, Vol. 44, No. 1., pp. 89-98, Spring 1997.

[4] J. Campanile, et. al., “GPS Acquisition Performance in the Presence of Jamming,” *Proceedings of the ION GPS-92*, Albuquerque, pp. 265-274, September 1992.

[5] K. T. Woo, “Optimum Semi-Codeless Carrier Phase Tracking of L2,” *Proceedings of the ION GPS-99*, Nashville, pp. 289-305, September 1999.

[6] M. K. Simon, et. al., *Spread Spectrum Communications Handbook*, New York, McGraw-Hill Inc., 1994.

[7] R. L. Peterson, *Introduction to Spread Spectrum Communications*, Prentice-Hall, NJ, 1995.

[8] S. Mahmood and C. Stewart, “An Analytical Study of Fast Acquisition Issues in GPS Exploitation Electronics Combat,” *Proceedings of the ION GPS-94*, Salt Lake, pp. 863-870, September 1994.